Screening Costly Information*

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June 8, 2024

Abstract

A monopolist offers a menu of quality-differentiated products. After observing the offer, a consumer can costly and flexibly learn which product is right for them. In the optimal menu, all types typically receive lower-than-efficient quality and distortions are, on average, more intense than under standard screening, even when no information is acquired. Profits are non-monotonic in the level of information costs, and the consumer may be better off when such costs are low than when information is free. We illustrate how an econometrician who ignores information acquisition might underestimate the level of inefficiency in the market.

*I am indebted to my advisor, Pietro Ortoleva, for his continued guidance throughout this project. I am grateful to Sylvain Chassang, Ana Luiza Dutra, Alessandro Lizzeri, Xiaosheng Mu, Franz Ostrizek, Rafael Parente, Wolfgang Pesendorfer, Pëllumb Reshidi, Denis Shishkin, Can Urgun, Leeat Yariv and Mu Zhang for very helpful suggestions and feedback. I also thank audiences at Princeton University, the University of California at Davis and the Young Economist Symposium for their helpful comments and discussions.

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1 Introduction

Consumers are often unsure about which product is right for them. In health insurance markets, for instance, potential buyers grapple with deciding which plan to contract, incurring large losses for selecting the wrong product (Abaluck and Gruber, 2011; Brown and Jeon, 2020; Handel and Kolstad, 2015). Frequently, additional information that could aid the agent's decision is available, but acquiring and processing it requires effort. In the insurance example, one could look at experience in previous years and research family history to forecast coverage needs, or compare how different plans cover various possible conditions. Whether the buyer expends effort to gauge their own preference or to assess how product characteristics match their personal taste, information acquisition is costly. We study how this costly information acquisition in demand affects equilibrium outcomes when a firm screens consumers.

This paper considers a model of vertical product differentiation in which buyers need to pay a cost to discover their type. Understanding equilibrium responses to information frictions is important for several reasons. First, extensive evidence shows that such frictions significantly affects consumer behavior.[1] Second, supply responses shed light on producers' incentives to aid or hinder consumer learning. Indeed, network provider and insurance company websites alike offer tools that help consumers compare plans, suggesting that sellers act to influence information acquisition. Finally, information frictions justify public interventions — in health care, governments may provide instruments for price tranparency (Brown, 2019), or standardize and simplify information on insurance plans (Ericson and Starc, 2016). The welfare consequences of these interventions cannot be ascertained without considering equilibrium effects. In fact, we show that helping buyers choose often reduces consumer welfare.

The Model As in Mussa and Rosen (1978), a monopolist (she) offers goods of different quality to screen different types of the agent (he). In standard screening, the buyer knows his taste for quality, but the seller does not. Here, we assume the agent's valuation is unknown to both players at the beginning of the game. Valuations may assume one of two values. After observing the menu of available goods, but before making a purchase decision, the buyer can costly and flexibly acquire information about how much he values quality; that is, the agent is rationally inattentive about his type. Information acquisition can be interpreted as the buyer learning either about his own preferences, or about product characteristics that affect his personal match value with the good. Because learning takes place after the seller makes her offer, the menu of goods will affect information choices: the monopolist can influence which information is acquired by choosing which contracts to provide. This interaction between contracts and costly learning is central to our analysis.

[1]See Abaluck and Gruber (2011), Brown and Jeon (2020), and Taubinsky and Rees-Jones (2018), for example for evidence on consumer behavior.

Results Information acquisition brings a new dimension to the monopolist's problem: contracts must induce the buyer to acquire the information the seller wants him to learn. This requirement implies two additional constraints to optimal screening. First, the principal must fine-tune goods' qualities for the agent to acquire specific information. If quality levels are far apart, selecting the wrong product implies large utility losses, and the buyer has considerable incentives to learn. If quality levels are close together, mistakes are less consequential, and the buyer will not choose to learn much. Thus, which information is acquired depends on the extent of product differentiation. Second, prices must be low enough for the consumer to buy. The more expensive products are, the higher the incentives for the buyer to learn: if he learns his value for quality is low, he does not purchase any goods, and thus he avoids paying too much for quality he does not enjoy. To discourage this behavior, the principal must offer price discounts.

We characterize the solution to the principal's problem and derive two key results. First, we show information acquisition adds to, rather than mitigates, the inefficiencies generated by information asymmetry alone. Quality distortions are both larger and more widespread than in standard screening. Distortions are larger in the sense that, on average, quality levels are further away from the efficient ones, when compared with the standard screening benchmark. They are more widespread because agents with all valuations typically receive below-efficient quality levels — in contrast to the standard screening result, where the buyer with the highest valuation is assigned efficient quality. The reason is that the consumer can deviate from the principal-preferred information decisions, learning what the seller does not want him to. The buyer receives rents not only due to the private information he actually bears, but also because of the information he could have obtained, and the seller distorts productive efficiency to save on such rents.

For an intuition, consider the following simple case. When acquisition costs are high enough, we prove no learning takes place, so information is symmetric in equilibrium. In standard screening, if information is symmetric, the seller offers a single good to the buyer, with efficient quality tailored to his valuation; prices are such that the consumer is indifferent between buying or not. Here, this offer does not work. If the agent is ex-ante indifferent between buying and not buying, then he has incentives to learn whether his true value is just a little smaller than his ex-ante belief. If he finds out his type is low, the agent does not purchase the good. To dissuade the buyer from learning, the seller gives the product a price discount and, to save on costs, degrades its quality. Thus, the contract is inefficient even when information is symmetric. This argument highlights the distinction between this model and standard screening: the key distortion here is the *threat* of obtaining information, rather than the presence of private information.

The second main result is that profits and consumer surplus are non-monotonic in the level of information costs. Profits decrease and consumer surplus often increases for low acquisition costs, whereas profits increase and consumer surplus decreases when costs are high. The reason is that the value of the agent's threat varies as costs change. When information costs are small, a fair amount of information is acquired in equilibrium. In that setting, the agent earns rents by threatening the principal not to learn as much as she wants. This threat gets more credible as acquisition costs rise and learning becomes harder. Thus, in that range, an increase in costs benefits the agent at the expenses of the principal. By contrast, when acquisition costs are high, information acquisition is very limited in equilibrium, and the agent extracts rents from the threat of learning more than the principal wants. Then, an increase in costs reduces the credibility of the threat, benefiting the seller and hurting the buyer.

Implications These results have several implications. The first is that information acquisition is fundamental for quantifying inefficiency. As previously mentioned, quality levels are lower on average in our model than they would be under standard screening. Thus, a researcher who estimates efficiency losses while ignoring the role of information acquisition would believe demand to be lower than it actually is. To rationalize that lower demand, the researcher would underestimate the taste for quality in the economy and, thus, the level of quality distortions. The estimation error could be substantial. We illustrate this error using the closed-form solution for the case in which both production and information costs are quadratic. We assume the researcher observes the quality of signed contracts but uses the results in Mussa and Rosen (1978) to quantify inefficiency. In a specific parameterization, the researcher may believe contracts are virtually efficient, whereas in reality, quality levels are about 45% lower than the optimal level.

Second, our results shed light on sellers' incentives to manipulate learning. These incentives vary with the level of information costs. If costs are low, it might be optimal for the principal to facilitate learning, by favoring transparency and providing tools that ease information acquisition. If acquisition costs are high, sellers may benefit from hiding product information and dissuading learning. Importantly, the seller may benefit from manipulating learning even if her action does not change information choices in equilibrium. Rather, changing the value of the buyer's threat is what drives the redistribution of surplus. This rationale complements insights from the literature on product obfuscation, in which firms dissuade consumer learning in order to profit against competitors.[2] In our model, the seller aims to reduce the buyer's strategic advantage and may even help buyers acquire information.

Finally, the non-monotonicity of consumer welfare has consequences for the design of policies that facilitate consumer learning. These transparency policies are often used in practice (Brown, 2019; Hackethal et al., 2012). The rationale supporting such interventions is based on how information costs affect consumers by causing choice mistakes: buyers fail to purchase the product that is right for them. Thus, when supply is fixed, consumer welfare increases if acquisition costs decrease. We show this argument may fail

[2]See Ellison (2005), Ellison and Wolitzky (2012), and Petrikaitė (2018).

when supply is allowed to respond. The seller benefits from more informed consumers, because she is then able to tailor products to consumer types. Hence, she is willing to provide incentives for the agent to learn and make fewer mistakes. When information costs are not too high, incentive provision more than compensates the buyer for his choice mistakes. Thus, transparency policies could have unintended equilibrium consequences, harming consumers, instead of benefiting them.

Related literature This paper contributes to the large body of work applying rational inattention models to different strategic interactions (Bloedel and Segal, 2020; Matějka and Tabellini, 2016; Ravid, 2020; Yang, 2019).[3] In particular, it relates to the literature on product market equilibrium when consumers are inattentive (Boyacı and Akçay, 2018; Cusumano et al., 2024; Hefti, 2018; Martin, 2017; Matějka and McKay, 2012). In these papers, firms offer a single good to agents who are rationally inattentive with respect to quality, price, or both. Closest to ours are Mensch (2022) and Mensch and Ravid (2022). Mensch (2022) studies the problem of selling a single good of fixed quality in the same information environment as this paper. Our framework differs in that here the principal offers multiple products, with endogenous quality levels, to induce or deter information acquisition. Thus, our model connects incentives for learning with product differentiation, a central contribution of this paper.

In concurrent work, Mensch and Ravid (2022) study a similar model of monopolistic screening with endogenous information acquisition, and also prove that quality distortions are more widespread than under standard screening. The papers differ in two dimensions. First, we provide additional results on the effect of these distortions on surplus. In particular, we show information costs may affect profits and consumers' surplus non-monotonically, with implications for firms' incentives to manipulate learning, as well as for the design of transparency policies. Second, we model information costs distinctly. While the two frameworks are equivalent when the state space is binary, their model allows for an uncountable state space and focuses on information costs that depend on the distribution of the buyer's posterior means.

Our paper complements a literature that studies how information acquisition interacts with mechanism design. Roesler and Szentes (2017) model a bilateral trade setting in which the buyer obtains information before the seller offers a menu. Similarly, Ravid et al. (2022) study the same setting when acquisition decisions and price setting occur simultaneously. These papers differ from ours in the timing of information acquisition: here, the monopolist commits to the menu beforehand, providing incentives for the information she wants to be acquired, which gives rise to the key distortion in our model. This timing distinction is critical: the main takeaway of Ravid et al. (2022) is that the consumer can be significantly better off when information is freely available, versus when it is extremely cheap. Our timing assumption reverses

[3] See Mackowiak et al. (2020) for a thorough review of applications of rational inattention by field.

that message, in that the buyer can be better off with costly information than when information is free. In section 5, we discuss this reversal and the role of timing.

Li and Shi (2017) and Guo et al. (2018) study information disclosure by a principal in a screening setting. In that setting, information is controlled by the seller, not by the buyer. Our work is related to an earlier literature on mechanisms with information acquisition (Bergemann and Välimäki, 2002; Crémer and Khalil, 1992; Shi, 2012; Szalay, 2009). We differ from these papers by focusing on multiple products and flexible information acquisition. The techniques we apply are connected with works on contracting with information design (Boleslavsky and Kim, 2018; Doval and Skreta, 2022; Georgiadis and Szentes, 2020; Ostrizek, 2020). Nevertheless, our problem is not one of information design, but rather of information acquisition. Finally, we are indebted to tools and ideas developed in decision problems under rational inattention, for example Caplin et al. (2022) and Matějka and McKay (2015). In particular, our solution to the information acquisition problem borrows the method developed by Caplin et al. (2022), based on concavification (Aumann and Maschler, 1995; Kamenica and Gentzkow, 2011).

2 A Model of Menu Design with Costly Information Acquisition

A monopolist (the principal, she) aims to sell indivisible goods to a potential buyer (the agent, he). She produces goods of different quality levels $q \geq 0$ at cost $c(q)$, where the cost function c is increasing, strongly convex and twice continuously differentiable.[4] The buyer's valuation for quality is $\vartheta \in \{\underline{\theta}, \overline{\theta}\}$, with $0 < \underline{\theta} < \overline{\theta}$. If an agent with valuation ϑ purchases a good of quality q at price t, he receives utility $\vartheta q - t$. The monopolist offers the buyer a menu of quality-price pairs. We assume throughout that $c(0) = c'(0) = 0$.

The model departs from traditional screening in two ways. First, information is symmetric at the beginning of the game: originally, ϑ is unknown to both players, who share a prior with mean μ. Second, before making his purchasing decision, but after observing the menu, the agent can acquire information about his valuation. Formally, he can choose an information structure $(\mathcal{S}, P)$, which consists of a set of signal realizations $\mathcal{S}$ and a function $P : \{\underline{\theta}, \overline{\theta}\} \rightarrow \Delta(\mathcal{S})$ that assigns a distribution over signals for each state. Although the agent is free to choose any information, learning is costly, as described below.

The timing of the game is as follows. The principal acts first, offering a schedule of quality-price pairs. After observing the offer, the agent decides which information to acquire. Upon observing a signal realization, the agent decides whether to buy one of the options from the menu or none — in which case he obtains a utility of zero. Importantly, the information choice is made only after the menu is observed and, hence, may depend on the menu of options offered by the monopolist.

[4] A function $f \in \mathcal{C}^2$ is strongly convex if $\min_x f''(x) > 0$.

Information and Acquisition Costs Because the agent's utility is linear, it depends on information only through the mean of posterior beliefs. Following standard practice, we associate each signal to the posterior mean it generates (Dworczak and Martini, 2019; Gentzkow and Kamenica, 2016). Formally, a signal realization is $\theta \in [\underline{\theta}, \overline{\theta}] \equiv \Theta$, and, upon observing signal θ, the buyer's expected payoff is $\theta q - t$. We refer to the realization of the agent's private information, θ, as his type or, abusing notation, his posterior. Then, any information structure is a distribution over realizations $F \in \Delta(\Theta)$ satisfying a Bayesian consistency constraint (Kamenica and Gentzkow, 2011). Because the state is binary, this constraint can be written as:

$$\mathbb{E}_F[\theta] = \mu. \tag{BC}$$

We assume information-acquisition costs are posterior-separable (Caplin et al., 2022). A convex function over posterior means, $H : \Theta \to \mathbb{R}$, exists, with $H(\mu) = 0$, and a scalar $k \geq 0$ such that the cost of an information structure F is defined as:

$$K(F) = k\mathbb{E}_F[H(\theta)].$$

This class of cost functions generalizes mutual-information-based acquisition costs, widely used in works on rational inattention, and encompasses almost all of the costs contained in the flexible information acquisition literature.[5] By obtaining information, the agent moves his posterior away from the prior. Heuristically, H defines this notion of distance between prior and posterior. The cost K reflects the expectation of this distance across all possible signal realizations according to information structure F. Note K is monotonic in the Blackwell order, in the sense that more informative structures are costlier. The parameter k scales the cost, and it is later used for comparative statics. We assume H is three times continuously differentiable in $(\underline{\theta}, \overline{\theta})$ and strongly convex.

Two subclasses of posterior-separable cost functions are of particular relevance. We say that a cost function has the unbounded marginal costs (UMC) property if marginally increasing the precision of a signal becomes arbitrarily costly as the signal becomes more precise. Formally, H is UMC if $\lim_{\theta \to \vartheta} |H'(\theta)| = \infty$, for $\vartheta \in \{\underline{\theta}, \overline{\theta}\}$. Mutual-information costs belong to this class. Conversely, we say a function is of bounded marginal costs (BMC) if $|H'|$ and H'' are bounded. A prominent example is quadratic cost, $H(\theta) = \frac{(\theta-\mu)^2}{2}$, which measures the expected reduction in prior variance obtained by observing information F.

[5] The standard formulation for posterior-separable information costs is in terms of posterior distributions. Because the state is binary, we can write it as a function of posterior means only. To embed mutual information, $H(\theta) = \sum_{\vartheta \in \{\underline{\theta}, \overline{\theta}\}} \left\{ \frac{|\vartheta - \theta|}{\overline{\theta} - \underline{\theta}} \log \frac{|\vartheta - \theta|}{\overline{\theta} - \underline{\theta}} - \frac{|\vartheta - \mu|}{\overline{\theta} - \underline{\theta}} \log \frac{|\vartheta - \mu|}{\overline{\theta} - \underline{\theta}} \right\}$. Part of the literature has been devoted to finding costs of acquisition that are alternative to mutual information, for example Hébert and Woodford (2020) and Pomatto et al. (2023). The cost specifications in these papers satisfy posterior-separability. Applications have either used Shannon entropy or other posterior-separable costs. Examples are Matějka and McKay (2015), Hébert and La'O (2020) and Yang (2019).

The Principal's Problem. We study the mechanism that maximizes expected profits for the monopolist. When choosing a mechanism, the seller takes into account that her offer affects the decisions of the buyer in two ways. First, it determines the buyer's choice given his information. In standard screening, the revelation principle guarantees that it suffices for the seller to offer one contract to each type of agent. However, here the agent type is determined by the information he decides to acquire, which is endogenous. As a consequence, the standard revelation principle does not apply directly in this setting. Nonetheless, the principal can restrict attention to menus of quality-transfer pairs $\mathcal{M} = \{q(\theta), t(\theta)\}_{\theta\in\Theta}$, in which one contract is offered to each possible posterior of the agent, including those that have zero probability in equilibrium.[6]

The second way in which the menu affects decisions is through the consumer's information choices. Because information is acquired after contracts are offered, acquisition depends on the terms of those contracts. By designing the menu, the principal indirectly controls information acquisition. Therefore, the seller maximizes profits knowing both which information will be acquired in equilibrium and how each type selects across contracts. Defining the outside option for the consumer as $C_o \equiv \{q(o), t(o)\} = \{0, 0\}$, we can write the principal's problem:

$$\max_{\mathcal{M}, F\in\Delta(\Theta)} \quad \mathbb{E}_F\left[t(\theta) - c\left(q\left(\theta\right)\right)\right]$$

$$\text{s.t.} \quad \theta \in \arg\max_{\omega\in\Theta\cup\{o\}} \theta q(\omega) - t(\omega), \quad \theta\in\Theta \tag{IC}$$

$$F \in \arg\max_{\mathbb{E}_G[\theta]=\mu} \mathbb{E}_G\left[\max_{\omega\in\Theta\cup\{o\}}\{\theta q(\omega) - t(\omega)\} - kH(\theta)\right]. \tag{IA}$$

The monopolist maximizes expected profits subject to two sets of constraints. The first one, IC, subsumes both traditional incentive compatibility and individual rationality constraints. It says that, given a posterior, $\theta \in \Theta$, the buyer chooses from the contracts in the menu and the outside option to maximize utility, obtaining interim rents $U(\theta) \equiv \max_{\omega\in\Theta\cup\{o\}}\{\theta q(\omega) - t(\omega)\}$. Note IC depends only on the offered menu, and not on which information is acquired. Thus, this constraint can be characterized following standard practice in mechanism design (Myerson, 1981). Given a distribution over types F, we denote the problem of maximizing expected profits subject to IC as the standard monopolistic screening problem (Maskin and Riley, 1984; Mussa and Rosen, 1978).

The information-acquisition constraint, IA, is the departure from standard screening. It says information must be a solution to the agent's acquisition problem. In that problem, the buyer chooses an information structure — namely, a Bayesian-consistent distribution of types — to maximize the expectation of

[6] We prove this revelation principle in the presence of information acquisition in Theorem 1 in Appendix B. The results is related to revelation principles for general type spaces as in Skreta (2006) and Hellwig (2010).

utility net of acquisition costs, $U - kH$. Costs are posterior-separable, and utility is given by interim rents: the maximum payoff of the contracting choice for each type realization. Finally, if multiple information structures solve the acquisition problem, we allow the principal to select his favorite; that is, we consider the principal-optimal equilibrium of the game.

3 Results

3.1 Simplifying the Principal's Problem

In this section, we turn the problem of the principal into a simple optimization. Two features of the problem connect contracting with information acquisition: we call them (i) marginal incentives for acquisition and (ii) the threat point. These properties allow for rewriting the principal's optimization into a tractable, finite-dimensional problem. By designing the menu, the principal provides incentives for information acquisition. (i) and (ii) characterize how the principal can provide such incentives: by controlling the marginal benefits of learning through product differentiation; and by adjusting the level of rents. In the discussion below, we explain these properties and their implications in constraining the space of contracts available to the principal.

Henceforth, we follow the standard practice of denoting menus as rent-quality pairs, $\{U(\theta), q(\theta)\}_{\theta\in\Theta}$. In the following discussion, we assume an optimal information has at most two posteriors. Any such information structure can be identified by the two posterior means in its support: $\operatorname{supp} F = \{\theta_L, \theta_H\}$, $\theta_L \leq \mu \leq \theta_H$. Because at most two posteriors are chosen, at most two contracts are signed with positive probability, $\{C_L, C_H\}$, with rents and quality levels $\{U_i, q_i\}_{i\in\{L,H\}}$, where $U_i \equiv U(\theta_i)$ and $q_i \equiv q(\theta_i)$. At the end of this subsection, Proposition 1 shows the optimal information structure is indeed binary. We characterize properties (i) and (ii) in terms of these equilibrium contracts.

Marginal incentives. The first property ties together product differentiation and the endogenously acquired information. To see how product differentiation affects the value of learning, consider the following heuristic argument. Recall that the consumer chooses information to maximize the expectation of utility net of costs, $U - kH$. Then, marginal net utilities $U' - kH'$ can be understood as the marginal value of information. But by the standard characterization of Myerson (1981), incentive compatibility implies $U'(\theta) = q(\theta)$, so the marginal value of information is affected by product quality.[7] Intuitively, agents benefit from information by choosing the contract that better reflects their tastes. As quality levels grow apart,

[7] A priori, it is not clear that the rent function U is differentiable at the relevant points in the support of F. This is formally shown in the proof of Lemma 1

this benefit increases, because purchasing the wrong contract has larger consequences. To induce a specific information strategy, the principal must fine-tune the quality of contracts to the information she wants to be acquired.

The marginal-incentives property makes this interplay between product differentiation and information acquisition precise. Roughly, it states that *marginal net utilities must be equated in the support of an optimal structure*. When this property fails, the buyer prefers to acquire information that is not predicated by the information structure, either because quality levels are too far apart and incentives for learning are excessive, or vice versa. If an information structure reveals a state, this argument works only partially. In that case, because the agent cannot learn more about the fully revealed state, an inequality of marginal utilities holds instead. The following result formalizes the discussion above and establishes the marginal-incentives constraint formally.

Lemma 1. *Let F and $\{U(\theta), q(\theta)\}_{\theta\in\Theta}$ satisfy IC and IA, with $\operatorname{supp} F = \{\theta_L, \theta_H\}$. Then, there is $\psi \in \mathbb{R}$ such that, for $i \in L, H$:*

$$\begin{aligned} q_i - kH'(\theta_i) &= \psi, \quad &&\textit{if } \theta_i \in (\underline{\theta}, \overline{\theta}) \\ q_L - kH'(\theta_L) &\leq \psi, \quad &&\textit{if } \theta_L = \underline{\theta} \\ \psi &\leq q_H - kH'(\theta_H), \quad &&\textit{if } \theta_H = \overline{\theta} \end{aligned} \tag{M}$$

Figure 1 illustrates this result graphically. The figure plots the net utility obtained at different posteriors, which are depicted in the horizontal axis. For each posterior, the agent chooses the best contract from $\{C_L, C_H\}$ or the outside option C_o, as shown in the bottom of the picture. The standard concavification argument implies that any optimal information structure, F, is represented by a segment tangent to the graph of $U - kH$.[8] We identify this segment with its slope ψ. The support of F are the types at which tangency happens — that is, $\operatorname{supp} F = \{\theta_L, \theta_H\}$ —, and the ex-ante utility of the agent is the height of the line segment at the prior mean. Note ψ plays an important role in our analysis. In particular, as we further emphasize later, ψ adjusts the value of information $U - kH$ to take into account the Bayesian-consistency constraint.

An immediate consequence of Lemma 1 is that, insofar as no posterior in the support of F is fully revealing, the difference between quality levels is pinned down by the marginal-incentives property:

$$q_H - q_L = kH'(\theta_H) - kH'(\theta_L)$$

Threat Point. The second property concerns the level, rather than the margin, of acquisition incentives: it is reminiscent of a participation constraint. Recall that the principal wants the agent to pick a contract from the menu she designs. However, the agent can always choose the outside option C_o after information

[8] For concavification in general, see Aumann and Maschler (1995) and Kamenica and Gentzkow (2011). For the argument applied to information acquisition, see Caplin et al. (2022).

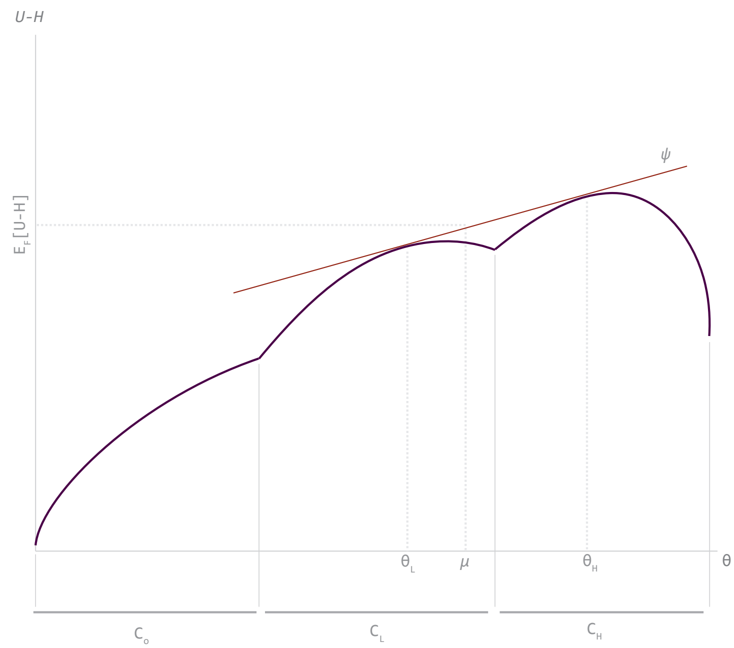
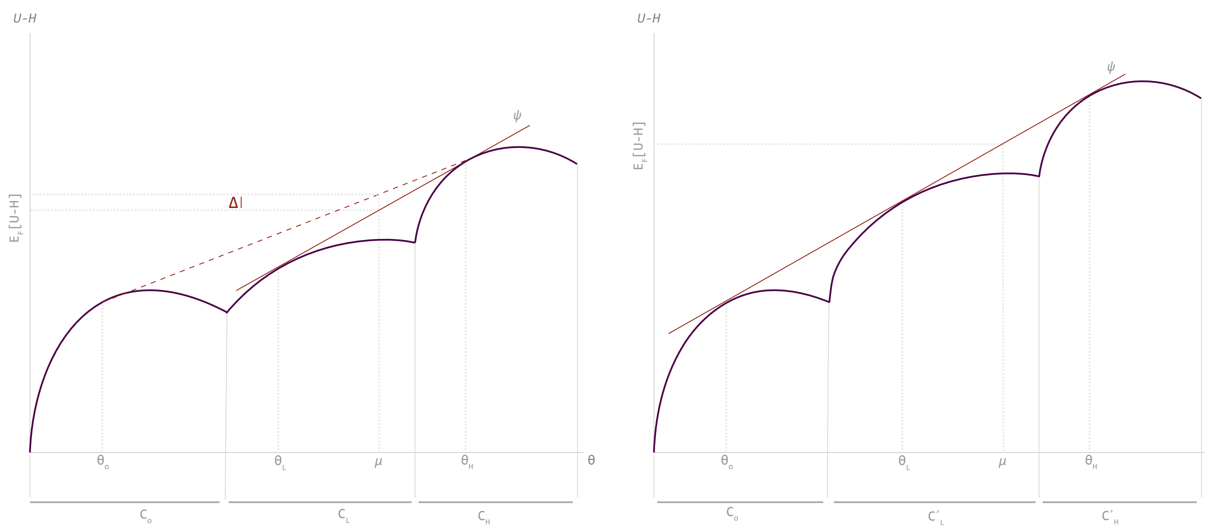


Figure 1: An Optimal Information Structure

realizes. He does so if he finds out that both contracts are too expensive given his information, that is, if he gets a signal that his type is too low. To guarantee participation, the principal must discourage the acquisition of any information that induces the agent to opt out of the menu. Figure 2a depicts a situation of non-participation. In that example, the agent rejects the principal's prescription, F, deviating to the information structure denoted by the dotted segment, with support $\{\theta_o, \theta_H\}$. If signal θ_o realizes, he opts-out of the menu. That deviation generates an expected gain of $\Delta > 0$ for the buyer. In the figure, F satisfies the marginal-incentives property, but the deviation is still attractive to the buyer: the seller must resort to a different set of tools to guarantee participation.

The principal can avoid such deviations by making the menu more attractive. In particular, she can affect the level of net utilities by reducing the prices of C_L and C_H, obtaining new contracts $\{C'_L, C'_H\}$, as can be seen in Figure 2b. By reducing transfers in each of these contracts, the principal raises rents — and, thus, net utility — for types that accept those contracts. Additionally, she increases the interval of types that participate in the menu, that is, the interval of posteriors choosing the outside option, C_o, shrinks. This change in contracts renders the prescribed information structure F optimal to the buyer by ruling out the deviation to θ_o.

To generalize this intuition, we identify a type that plays a central role in the opting-out deviations,



(a) A profitable deviation

(b) No profitable deviation

Figure 2: Net Utilities and the Threat Point

given the quality levels and prescribed information: we call it the threat point. The threat point is the posterior that maximizes the ex-ante value of purchasing C_o for the buyer. Recall that the value of information is given by the net utility $U - kH$. By trying to maximize this value, the buyer must take into account the Bayesian consistency constraint. To do so, the buyer maximizes the adjusted value of information $U - kH - \psi\theta$. The threat point, $\theta_o(\psi)$, is the posterior that maximizes the agent's adjusted value of information subject to not participating in the mechanism:

$$\theta_o(\psi) \equiv \arg\max_{v \in \Theta} \{-kH(v) - \psi v\}.$$

Lemma 2. *Let $F, \{U(\theta), q(\theta)\}$ solve the principal's problem with* $\operatorname{supp} F = \{\theta_L, \theta_H\}$*. Furthermore, assume ψ is as in Lemma 1. Then, the following holds:*

$$U_i - kH(\theta_i) - \psi\theta_i = -kH(\theta_o(\psi)) - \psi\theta_o(\psi) \quad \textit{for all} \;\; i \in \{L, H\} \tag{TP}$$

As in Figure 2b, Lemma 2 shows the relation between contracts in the support and the threat point is tight: the consumer is exactly indifferent between learning $\theta_o(\psi)$ and following F. The expression on the left-hand side is the adjusted value of obtaining posterior θ_i, that is in the support of information structure F, prescribed by the principal. The right-hand side is the adjusted value of learning the threat point. This indifference implies the threat point determines the level of rents, which in turn pins down qualities, because rents and quality are linked byincentive compatibility. A higher threat point reduces quality and rents. Formally, if the principal wants to implement information structure $\operatorname{supp} F = \{\theta_L, \theta_H\}$

such that no type is fully revealing of a state, quality levels must satisfy:

$$q_i = kH'(\theta_i) - kH'(\theta_o(\psi)), \quad i \in \{L, H\}.$$

Together, marginal incentives for acquisition and the threat point tightly constrain the set of contracts the principal can choose from. Marginal incentives for acquisition tie the differences in quality levels for any two contracts to the prescribed information structure, whereas the threat point fixes the quality level. Given the choice of information, the only instrument remaining for the principal is the choice of the threat point or, equivalently, the dual variable ψ.

A simpler problem Lemma 1 and Lemma 2 show M and TP are constraints to the principal's optimization. The following result proves they are the only constraints the principal must satisfy in addition to Bayesian consistency, BC. We say two optimization problems are equivalent if they have the same value and the solution to one can be used to construct a solution to the other.

Proposition 1. *The principal's problem is equivalent to:*

$$\max_{\{\theta_i, U_i, q_i\}_{i\in\{L,H\}}, \psi} \Big\{ \sum_{i\in\{L,H\}} p_i^F \left[\theta_i q_i - c(q_i) - U_i\right] : M, \;\; TP \quad and \quad BC \Big\}, \tag{P}$$

where p_i^F is the probability of θ_i under $\operatorname{supp} F = \{\theta_L, \theta_H\}$.

Furthermore, this problem has a solution.

Proposition 1 shows the principal's problem can be greatly simplified to a finite dimensional optimization with equality and inequality constraints. The objective function is the principal's profits rewritten as surplus minus rents, as standard, and assuming F is at-most-binary. Essentially, the result states that the principal can focus on binary information structures and on equilibrium variables: posteriors and contracts that are signed with positive probability in equilibrium. In the original principal's problem, the seller must take into account the off-equilibrium behavior of the buyer: the information they could have chosen to obtain and the contracts they would pick if they acquired different information. In Proposition 1, instead, the principal only needs to track the slope ψ, which, in addition to equilibrium variables, guarantees that the agent will follow the plan of action she prescribes.

For an intuition, note our discussion thus far has proved that M and TP must hold in a solution to the principal's problem. Thus, provided the restriction to binary distributions, the profits attained in the problem in Proposition 1 are at least as large as the principal's profits. The majority of the proof consists in showing the converse, namely, that any solution to the problem above can be extended to satisfy IC and IA without loss of profits. This result is obtained in several steps. First, we show contracts are incentive compatible in $\{\theta_L, \theta_H\}$, which is guaranteed because M implies quality is monotonic, and TP allows us to compare interim rents. In fact, IC holds strictly: the principal needs to provide more incentives for the agent to acquire information, than for him to simply reveal it. TP also implies individual rationality. Then, we show we can extend contracts to satisfy IC to posteriors that have zero probability in equilibrium. Finally, a concavification argument guarantees IA is also satisfied.

A tractable problem Proposition 1 delivers a tractable optimization problem suitable for applications. For an illustration, consider the case in which H is UMC, so the optimal information structure contains no fully revealed state. As a consequence, M holds with equality, allowing us to write quality, q_i, as a function of θ_i and ψ. Similarly, one can use the threat-point property, TP, to solve for interim rents, U_i, and Bayes-consistency can be used to write p_i^F as a function of $\operatorname{supp} F$ and μ. Substituting q_i, U_i, and p_i in the seller's optimization, we obtain the following remark:

Remark. *When H is UMC the problem of the principal can be rewritten as*[9]*:*

$$\max_{\theta_L\leq\mu\leq\theta_H,\psi}\sum_i \frac{|\theta_j-\mu|}{\theta_H-\theta_L}\Big\{\theta_i\left(kH'(\theta_i)-kH'(\theta_o(\psi))\right)-c\left(kH'(\theta_i)-kH'(\theta_o(\psi))\right)$$
$$-\left[kH(\theta_i)-kH(\theta_o(\psi))+\psi\left(\theta_i-\theta_o(\psi)\right)\right]\Big\}.$$

The problem above is a simple optimization over three variables. Next, we use P to characterize general properties of the optimal menu and of the surplus, which are the main results of the paper. We describe these results and discuss how they are guided by the two key forces described above: the marginal-incentives property, M, and the threat point, TP. It is worth noting that, as k converges to zero, the principal's problem converges to the standard screening problem in which the consumer is fully informed.

3.2 Menu Design and Information Acquisition

This section focuses on the efficiency of optimal contracts. We compare these contracts with the first-best allocations and with the ones obtained in standard screening. This comparison is not straightforward. Here, the agent is uninformed to begin with, and all the information is obtained endogenously by acquisition choices. By contrast, in the standard screening model, information is exogenous. To evaluate quality distortions, we keep information constant across models: we first solve for the optimal information structure in our model and then take it as the exogenous information in a standard screening problem.

The efficient (or first-best) quality for type θ_i is the quality that maximizes the production surplus for that type, namely, q_i^{f} such that $c'(q_i^{\mathrm{f}})=\theta_i$. Under symmetric, exogenous information, each type of buyer is assigned a contract with efficient quality. In the second-best problem, when information is exogenous but asymmetric, the principal wants to dissuade the high-type agent from purchasing contracts aimed toward

[9] The simplified optimization problem when costs are not UMC can be found in the proof of Proposition 2 in the Appendix.

the low type. The optimal way for her to achieve that goal is to make the low contract less attractive. Thus, in the standard screening model, the principal underprovides quality to the low type, $q_L^s < q_L^{\mathrm{f}}$. On the other hand, the quality assigned to the high type, $q_H^s = q_H^{\mathrm{f}}$, is efficient. Note that the sole driver of this distortion is private information: the principal needs to avoid attracting high types to the low contract, because she is uninformed about consumer's value.

For any information structure F and menu of contracts with quality levels q, $\tau(F,q) \equiv \mathbb{E}_F[\theta - c'(q(\theta))]$ denotes the expected wedge: it measures how distorted from efficiency the quality levels in the menu are. In particular, $\tau(F,q^{\mathrm{f}}) = 0$ because the first-best menu maximizes production surplus, and $\tau(F,q^s) > 0$. We now show that when information is endogenously acquired, in contrast to the pure screening case, both types receive below-efficient quality. Additionally, the wedge is larger than in the second-best solution.

Proposition 2 (Distortion Patterns)**.** *Let $F, \{U, q^*\}$ solve the principal's problem. Then:*

Quality is underprovided. $c'(q_L^*) < \theta_L$ *and* $c'(q_H^*) \le \theta_H$

Aggregate distortions. $\tau(F,q^s) \le \tau(F,q^*)$

All inequalities are strict if either (i) k is high enough; (ii) $k > 0$ and H is UMC; or (iii) whenever $\operatorname{supp} F = \{\mu\}$.[10]

Proposition 2 shows that in the presence of information acquisition, distortions also happen at the top; moreover, they are larger than under standard screening. To understand the result on the wedge, note that our model adds a new distortion to the screening problem: the threat from the agent to obtain information. The buyer could obtain information on whether his type is low and opt-out of the menu if it is the case. By the threat-point property, the principal must reduce prices to guarantee no such deviation is profitable for the buyer. The efficient way to reduce prices, from the production perspective, is to also degrade quality. As a result, aggregate distortions are higher, on average. For an interpretation of the result, consider the case in which production costs are quadratic, $c(q) = \frac{q^2}{2}$. In that case, the increase in the wedge means that the average quality provided under endogenous information is lower than the average quality provided in the second-best, when information is exogenous: endogeneity increases the misallocation of qualities to types.

To show that both qualities are underprovided, it is helpful to think of the principal's problem P as an information design problem, given the threat point θ_o — or equivalently, the dual variable ψ. The principal wants to maximize the expectation of the profits obtained at posterior θ, $L(\theta) = \theta q(\theta) - c(q(\theta)) - U(\theta)$. Optimal information design requires that the principal's profit is concavified, which in turn implies

[10] Lemma 5 in Appendix A shows that $\overline{k}$ exists such that $\operatorname{supp} F = \{\mu\}$ whenever $k \ge \overline{k}$.

that marginal profits of changing posteriors are equalized at any point in the support. By the marginal-incentives property, an increase in posteriors leads to an increase in quality, and it's impact in profits is proportional to the wedge. Heuristically:

$$L'(\theta) = \underbrace{(q(\theta) - U'(\theta))}_{\text{0 by IC}} + (\theta - c'(q(\theta)))\, q'(\theta)$$

Concavification then implies that the wedges have the same sign. Because, on average, the wedge is positive, a distortion at the bottom begets a distortion at the top. As discussed in the Introduction, the inefficiency persists even when information is symmetric: distortions are determined by the threat of information acquisition, rather than by private information itself.

Note the inequalities in Proposition 2 are not always strict. In particular, for low k, acquiring complete information can be optimal. In this case, it is always profit maximizing for the principal to provide the same quality levels as under standard screening — although not the same prices. However, for sufficiently large costs, the inequalities are strict and the difference between this model and standard screening is sharp. Similarly, UMC cost functions guarantee that information cannot be fully acquired and thus that inequalities are always strict. The remainder of this section illustrates these results with a special case in which we completely characterize the optimal solution.

3.2.1 Quadratic Costs

Assume $c(q) = \frac{q^2}{2}$ and $H(\theta) = \frac{(\theta-\mu)^2}{2}$. Under this specification, the cost of acquiring an information structure is proportional to the reduction in prior variance obtained by observing this information.[11] As a function of k, we denote the equilibrium information structure as $F(k)$ and equilibrium quality as $q^*(k) = \{q_L^*(k), q_H^*(k)\}$. We also denote $q^s(k)$ as the second-best quality obtained when the information is exogenously set at $F(k)$. The next proposition characterizes $F(k)$ and compares $q^*(k)$ with $q^s(k)$.

Proposition 3 (Quadratic Costs)**.** *Under quadratic costs, $\underline{k} \le \overline{k}$ exist such that the optimal structure $F(k)$ satisfies:*

$$\operatorname{supp} F(k) = \begin{cases} \{\underline{\theta}, \overline{\theta}\}, & \text{if } k < \underline{k} \\ \{\omega(k), \overline{\theta}\}, & \text{if } \underline{k} \le k < \overline{k} \\ \{\mu\}, & \text{if } k > \overline{k} \end{cases}$$

where ω is a strictly increasing, continuous function with $\omega(\underline{k}) = \underline{\theta}$. $F(k)$ is unique except, possibly, at $\overline{k}$. Moreover, $q_H^*(k) = q_H^{\mathrm{f}}(k) = \overline{\theta}$, for all $k < \overline{k}$ and $q_L^*(k) \le q_L^s(k)$.

[11] Specifically, $K(F) = \frac{k}{2}\mathbb{V}_F[\mathbb{E}[\vartheta|\theta]]$. Then, by the Variance Decomposition Formula, $K(F) \propto \mathbb{V}[\vartheta] - \mathbb{E}_F[\mathbb{V}[\vartheta|\theta]]$.

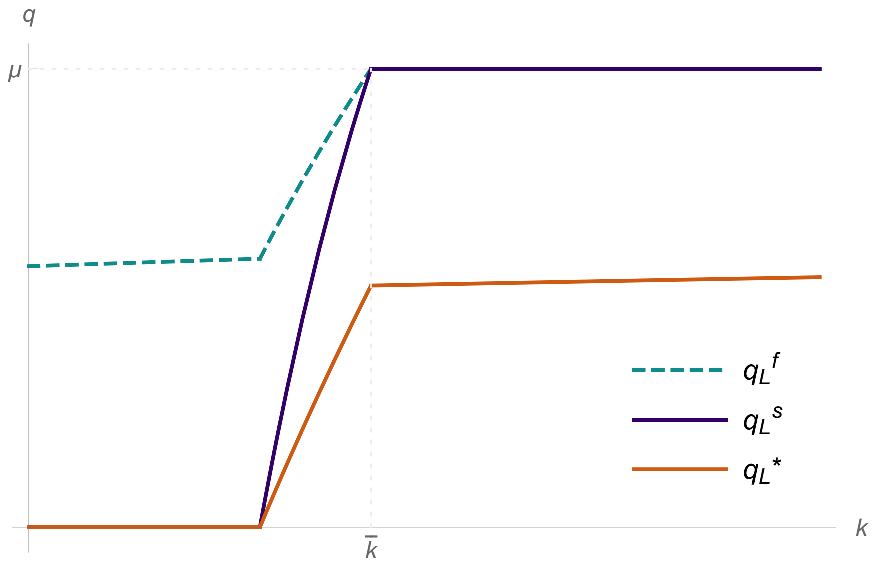


Figure 3: Low Quality Under Quadratic Costs

Notes: This figure plots the quality of the low type under quadratic costs as k varies, q_L^*. For each k, we use the information structure $F(k)$ to solve for the first- and second-best contracts q_L^{f} and q_L^s. For ease of visualization, we start the horizontal axis from $k > \underline{k}$.

In words, $F(k)$ has a simple form: it contains three regions depending on k. For low k, full information is acquired and the state is revealed. For high k, no information is acquired at all. For intermediate levels of costs, the information consists of one posterior that fully reveals the high state, $\overline{\theta}$, and of a low posterior that is partially informative. In that range, as k increases, the low posterior is increasing; that is, the precision of the low signal deteriorates monotonically. Although the specific form of this optimal structure is special, the existence of $\overline{k}$ such that no information is acquired for $k \geq \overline{k}$ holds for any information and production costs. Similarly, the existence of $\underline{k}$ such that the states are perfectly revealed for $k \leq \underline{k}$ is guaranteed by any BMC information costs.[12]

The high state $\overline{\theta}$ is fully revealed because of the marginal-incentives property, M. By M, quality levels must be fine-tuned not to give the agent incentives to make one of his posteriors more precise. However, $\theta_H = \overline{\theta}$ is as precise as a posterior can be, so the principal does not have to worry about that kind of deviation. Thus, the seller is free to provide quality to the high agent that is discontinuously larger than what she could offer if the high posterior were all but perfectly informative. She seizes that opportunity, providing the high type with his efficient quality. As k grows, maintaining full revelation at the top becomes costlier and the seller degrades the precision of the low posterior, increasing the quality of the product sold to the low type.

Because the high type receives efficient quality, all the distortions in this example come from the con-

[12]These results are proved in Lemma 5 and Lemma 6 in Appendix A.

tract given to the low type. Figure 3 plots $q_L^*(k)$ and compares it with $q_L^s(k)$ and $q_L^{\mathrm{f}}(k)$. Because production costs are quadratic, $q_L^{\mathrm{f}}(k) = \omega(k)$. The distance between q_L^{f} and q_L^s is the standard screening distortion. It stems from asymmetric information exclusively; therefore, it vanishes for $k \geq \overline{k}$, when information is symmetric in equilibrium. The difference between q_L^s and q_L^* is the distortion given by information acquisition: it is a result of the agent's threat of acquiring information that is not prescribed by the principal. This difference does not disappear when information is symmetric: the distortion persists and only vanishes asymptotically, as k approaches infinity and the agent loses his ability to threaten the principal.

3.3 Nonmonotonic Surpluses

We now turn to discussing how the level of acquisition costs, k, affects profits and consumer surplus. Several mechanisms are at play. First, information has a productive role in this model, helping agents match with contracts of appropriate quality, generating higher surplus. Thus, higher information costs, by constraining the production frontier, could have a negative effect on surpluses. Second, acquired information is costly and must be paid for. To the extent that these expenses vary with the level of information costs, this level affects the surpluses. Finally, costs affect the balance of power in the principal-agent relationship in two ways. One is direct, as k maps into the value to the buyer of deviating from the prescribed information strategy. Because, in equilibrium, he must be compensated for not deviating, k affects the division of surplus. The other is indirect: the level of costs affects which information is acquired and therefore affects information asymmetry. Proposition 4 below describes the end result of all these mechanisms on profits and consumer surplus. The following assumption is relevant.

Assumption 1. *At least one of the following holds:*

1. *H is BMC;*
2. *$(\overline{\theta} - \underline{\theta})^2 \geq (\overline{\theta} - \mu)\overline{\theta}$.*

Assumption 1 describes a class of markets that includes all bounded marginal costs of information. Condition 2 in the assumption does not depend on information costs. It requires, instead, that, under full information, the second-best contract excludes the low type, that is, $q_L^s = 0$. We define consumer surplus as the ex-ante net utility of the agent under the optimal information structure: $\mathbb{E}_F[U(\theta) - kH(\theta)]$.[13]

Proposition 4 (Nonmonotonicity)**.** *The principal's profits increase and consumer's surplus decreases for sufficiently large k. The principal's profits decrease for sufficiently small k. Under Assumption 1, consumer's surplus increases for sufficiently small k.*

[13] The same comparative statics in Proposition 4 holds for gross utility $\mathbb{E}_F[U(\theta)]$.

Proposition 4 shows that as the level of acquisition costs changes, profits respond non-monotonically. Under Assumption 1, consumer surplus is also non-monotonic and roughly in the opposite way. Profits first decrease, when information costs are low, and then increase when k is high. Under Assumption 1, the opposite holds for consumer surplus.

The intuition for this result is that changing k affects the value of the agent's deviations. When k is small and thus the prescribed information structure is particularly revealing, the most valuable threat for the agent is acquiring too little information. As k increases, that threat becomes more valuable: the agent has fewer incentives to learn as learning gets costlier. This increase initially works in his favor and at the expense of the seller. By contrast, when k is very high and the prescribed information structure is extremely opaque, the most valuable threat is learning too much. However, as costs grow, that deviation becomes less credible. Then, further increases benefit the principal at the cost of the buyer. To see why it is the threat that drives the result, rather than the productive role of information, consider again the case of quadratic costs. Recall that, in that case, the agent acquires a fully informative information structure and contracts have the second-best quality for $k \leq \underline{k}$. Over this interval, production is just as efficient as in the second-best, so the fact that profits are falling — and consumer surplus increasing — over that range is unrelated to the value of information. A similar rationale works for $k \geq \overline{k}$. Assumption 1 guarantees that this intuition carries over to more general information cost functions. By the same token, the argument above implies that total surplus moves in the opposite direction of consumer surplus for extreme values of k.

Implications The non-monotonicity result has two main implications. First, it sheds light on firms' incentives to aid or dissuade consumer learning. In many settings, a seller can help or obstruct consumer learning by making testing, experimenting, or having access to valuable information easier or harder. For example, insurance and mobile phone provider websites often help customers find plans that best suit their needs. When should we expect sellers to hinder or help information acquisition? If the seller has a limited ability to affect the level of acquisition costs, then she would prefer to hamper consumer learning if those costs are sufficiently large, but to facilitate learning if costs are low. Here, obfuscation is profitable when the agent threatens to learn more than what the monopolist desires. For a given level of costs, preventing this threat would imply distorting production surplus and providing price discounts. On the other hand, by making acquisition harder, the seller discourages learning, achieving the same goal with smaller efficiency losses. When the agent threatens to learn less than the seller wants, facilitating learning has the same effect.

Second, the non-monotonicity of consumer surplus suggests policies that facilitate consumer learning may not benefit consumers. Transparency policies are relatively popular in markets for complex goods.[14] For example, New Hampshire provides the public with the HealthCost website, which is a price information tool for health-care costs (Brown, 2019). Importantly, this website allows consumers to compare out-of-pocket medical procedure costs across providers, taking into account personal information, as their insurance carrier and zip code. Similar tools are also available in many other states (Brown, 2019). The rationale for these policies seems to be that reducing information costs will help consumers make better decisions and, thus, increase their welfare. However, this rationale ignores equilibrium effects, which may reverse the intended effects of such intervention. In particular, under Assumption 1, the nonmonotonicity of consumer surplus implies that some level of opaqueness in the market is optimal for consumers.

3.4 Estimating Efficiency Losses in Monopolistic Screening

A sizable literature empirically studies nonlinear pricing models in several markets, from cable television to health care; see Perrigne and Vuong (2019) for a recent survey. Applying these models to real data allows researchers to measure quality degradation (Crawford et al., 2019; Crawford and Shum, 2007), quantifying the extent to which firms with market power generate inefficiencies in environments with asymmetric information. This literature applies the first-order conditions of the principal (Luo et al., 2018) to identify variables of interest, therefore relying heavily on the model specification.

Our results suggest that, by ignoring information acquisition, this approach may mismeasure the true level of quality degradation. To make this point concrete, we focus on the simplest possible case. We assume contract quality levels $q > 0$ are observable and costs are quadratic. Additionally, we assume costly information acquisition exists in reality, but a researcher ignores it and tries to estimate quality degradation using the standard screening results of Mussa and Rosen (1978). The researcher observes the distribution of signed contracts and their quality and knows production costs are quadratic. Her goal is to estimate the distribution of types, F, in the economy. Given F, the level of quality distortion can be measured by the expected wedge, which reads $\tau(q,F) = \mathbb{E}_F[\theta] - \mathbb{E}_F[q]$ in this case. Under quadratic costs, the wedge is the difference between efficient and assigned average quality. Let $\hat{\tau}$ be the estimated wedge given what the researcher observes.

Proposition 5. *Under quadratic costs, $\hat{\tau} \leq \tau(F,q)$.*

Proposition 5 shows that, if costs are quadratic, a researcher would always underestimate the expected gap between efficient and realized quality and thus would overestimate efficiency. This result follows a

[14] See Brancaccio et al. (2017) for an example in finance, Brown (2017, 2019) for health care, and Hackethal et al. (2012) for cellular plans.

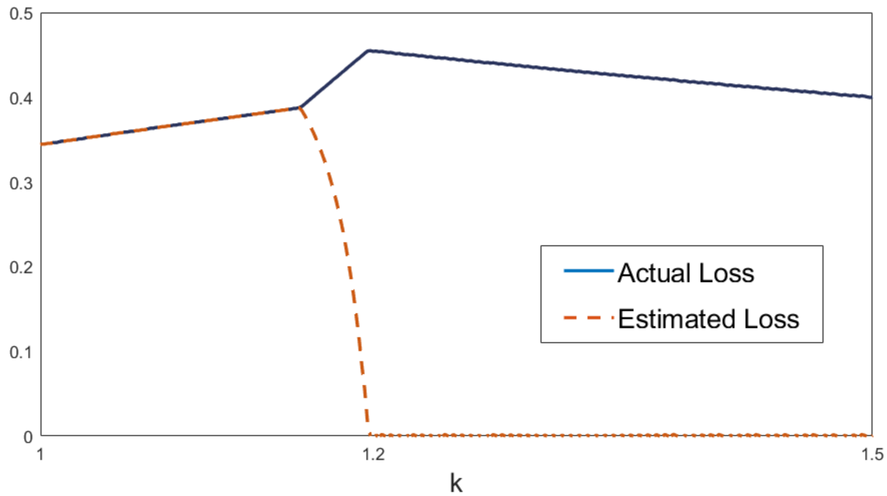


Figure 4: Underestimating Inefficiency

Notes: This figure plots real and estimated quality degradation as k varies. Degradation is measured as the percentage deviation between expected quality and the efficient one: $\frac{\tau(F,q)}{\mu}$. The picture corresponds to parameters $\{\underline{\theta}, \mu, \overline{\theta}\} = \{.2, .5, 1\}$.

simple argument. In a binary setting, F consists of three variables: θ_L, θ_H, and the frequency of high types, which we call p_H^F. By observing the market share of each good, and assuming efficiency at the top, the researcher can recover p_H^F and θ_H. Proposition 3 guarantees these estimates are correct. However, issues arise when the researcher tries to estimate θ_L using q_L. By Proposition 3, q_L is smaller than the quality that would be assigned to the true low type θ_L in standard screening. To rationalize the low level of quality, the researcher would have to believe the low type is lower than θ_L. Their estimate of θ_L would be the threat point $\hat{\theta}_L = \theta_o(\psi)$, leading the researcher to underestimate the mean $\mathbb{E}_F[\theta]$, and, consequently, the wedge.

Our solution to the quadratic example allows us to quantify the discrepancy between the real efficiency loss and the one estimated by the researcher, as illustrated in Figure 4. We solve our model for different levels of information costs and measure the relative level of quality degradation, given by $\frac{\tau(F,q)}{\mu}$, represented by the full curve in the figure. Then, for each k, we reproduce the estimation procedure we described above, assuming the researcher observes the equilibrium quality obtained in our model, thus leading to the estimated level of degradation given by the dashed curve. For low enough k — for example, k close to 1 —, the estimate is correct, because $\theta_o(\psi) = \theta_L$. When costs are high enough that no information is acquired, only one contract is signed in equilibrium and estimated losses are 0, but real losses are positive and often large. Crucially, even for intermediate cases, when two contracts are signed in equilibrium, the researcher could severely underestimate losses. For example, when $k \approx 1.2$, the researcher would estimate minimal losses when quality levels are in fact about 45% lower than efficient.

4 Discussion

Timing We studied the contracting problem when the agent acquires information after the principal offers the menu. This assumption is realistic: in a variety of settings, the menu of available goods is fixed, and consumers can choose to acquire information after observing it, according to their own timelines. Examples are health care and online shopping, where the available goods and terms of trade are typically readily available and easily observable. In other common environments, however, information can be acquired before the menu is offered. In particular, when observing the terms of trade depends on an action by the buyer — for example, contacting a dealer for a financial-asset quote, or reaching out to a vendor to learn about prices —, information acquisition can happen before the seller's offer is fixed. This possibility can also hold when the producer is able to make exploding or timed offers. Roesler and Szentes (2017) and Ravid et al. (2022) study bilateral trade models in which information acquisition happens before and simultaneously to the design of the mechanism, respectively. Together, their work and this paper shed light on the importance of the timing assumption in determining the outcome of mechanism design under information acquisition.

Our model complements Ravid et al. (2022) by showing our timing assumption reverses their main takeaway. The key message in Ravid et al. (2022) is that, when information is acquired simultaneously to the design of the mechanism, the buyer may be substantially better off having access to free information than being able to purchase the same information at a low cost. When buyer and seller decide at the same time, the corresponding optimal mechanism fails to induce sufficient information acquisition even when costs are arbitrarily small. As the buyer foregoes some amount of information, the authors show the price at the optimal mechanism is higher than it would be if the buyer learned more thoroughly. As a consequence, both the consumer and the producer are worse off than in the full-information equilibrium that could arise when information is free.[15] As previously discussed, we obtain the opposite result: from the point of view of the consumer, low information-acquisition costs can be strictly better than being able to acquire information for free. The reason is that the principal, acting first, chooses to compensate the agent to acquire surplus-enhancing information when doing so is inexpensive. That compensation works in the buyer's favor and may increase consumer surplus.

Posterior-Separability We assumed information costs to be posterior-separable (Caplin et al., 2022). This has three related implications. First, under posterior-separability one can restrict attention to binary information structures. This is a consequence of the linearity of posterior-separable costs with respect to the

[15] In fact, they are worse off than under any equilibrium of the costless information economy.

posterior distribution. Second, and more importantly, posterior-separable costs allow for a characterization of implementability as two properties: marginal incentives and the threat point. In our model, we leverage tools from information design to guarantee that these properties characterize the constraint set of the principal's problem, which cannot be done for general costs. In particular, deviations to non-participation, which for posterior-separable costs are parameterized by a single posterior — the threat point — would depend on the whole deviating distribution of posteriors, compromising the tractability of this approach.

The third implication of posterior-separability is that, conditional on the threat point, the problem of the principal becomes a regular information design optimization. Because our argument for establishing underproduction at every point in the support relies on concavification, it cannot be readily extended for general cost functions. Nevertheless, whenever the principal decides to induce no information acquisition, the underprovision result generalizes. The key assumption, that is satisfied by posterior-separable costs, is that the consumer can move beliefs from the prior at zero marginal costs. To formalize this result, we restrict attention to binary information structures, and extend the model in the text for general cost functions. Concretely, if $\{\theta_L, \theta_H\} = \operatorname{supp} F$, we let $K(F) = h(\theta_L, \theta_H)$, where h is decreasing in θ_L and increasing in θ_H.

Assumption 2. *h is continuously differentiable and $\frac{\partial h}{\partial \theta_L}(\mu, \mu) = \frac{\partial h}{\partial \theta_H}(\mu, \mu) = 0$*

Proposition 6. *Under Assumption 2, if the principal wants to induce no information, quality is underprovided: $c'(q^*) < \mu$.*

The rationale for this result parallels the one for posterior-separable costs. In standard screening, if information is symmetric, the seller offers a single good to the buyer, with efficient quality for his valuation and prices are such that the consumer is indifferent between buying or not. Here, this offer does not work. If the agent is indifferent between buying and not buying when uninformed, he always has access to a cheap enough experiment that may tell him that his type is slightly lower than the ex-ante type. Therefore, to guarantee that the buyer acquires no information, the principal gives a price discount and optimally distorts quality accordingly. This suggests the force for underproduction is still present even when information costs are not posterior-separable. In particular, the result that distortions exist even under symmetric information does not depend on details of information costs.

Appendix A: Auxiliary Results and Proofs

We start proving an auxiliary result. The information acquisition problem can be written in terms of rents as:

$$\begin{aligned} \max_{G\in\Delta(\Theta)} \quad & \mathbb{E}_G\left[U(\theta)-kH(\theta)\right] \\ \text{s.t.} \quad & \mathbb{E}_G[\theta]=\mu \end{aligned} \tag{1}$$

Lemma 3. *Problem 1 has a solution. $F\in\Delta(\Theta)$ solves it if and only if it satisfies BC and there exists $\psi\in\mathbb{R}$ such that:*[16]

$$\operatorname{supp} F\subseteq \arg\max_{v\in\Theta}\{U(v)-kH(v)-\psi v\} \tag{2}$$

Proof of Lemma 3

By strict convexity of c, it is without loss of generality to assume (q,t) is bounded. We start by proving existence of a solution. We then proceed to show necessity and sufficiency of condition 2.

Existence. Because contracts are bounded and Θ is compact, interim rents U are bounded by definition. Thus, $U-kH$ is a bounded function and the objective function is trivially continuous with respect to F in the weak topology. Additionally, because Θ is compact, $\Delta(\Theta)$ inherits compactness in the topology of weak convergence, by an application of Prokhorov's theorem. Finally, the set of F satisfying BC is closed under weak convergence, implying that the constraint set is a closed subset of a compact space, being itself compact. As a consequence, under the topology of weak convergence, problem 1 is one of maximizing a continuous function over a compact set and, therefore, has a solution.

Necessity. That BC is necessary is trivial, as it is a constraint in the problem. For 2, Start by defining the Lagrangian:

$$L(F,\psi)=\mathbb{E}_F\left[U(\theta)-kH(\theta)-\psi\theta\right]$$

Notice that, as $\mu\in(\underline{\theta},\overline{\theta})$, it is an interior point of the set $\{y\in\mathbb{R}:\mathbb{E}_F[\theta]=y \quad \text{for some} \quad F\in\Delta(\Theta)\}$. Given that, Luenberger (1997), Chapter 8, Problem 7 proves that if F solves 1, then there exists ψ such that $F\in\arg\max_{G\in\Delta(\Theta)} L(G,\psi)$. We now prove that this implies 2.

[16] This result is a consequence of the Lagrangian Lemma in Caplin et al. (2022). We adapt it to our framework and provide a short proof.

Define $\chi \equiv \arg\max_{\theta\in\Theta}\{U(\theta) - kH(\theta) - \psi\theta\}$. Assume, so as to find a contradiction, that $v \in \operatorname{supp} F$ exists such that $v \notin \chi$. It is immediate that F cannot put positive weight outside of χ. Then, assume v is a continuity point of F. That implies there is a neighborhood of v, N_1, such that $N_1 \in \operatorname{supp} F$. However, because $U - kH$ is continuous, there is another neighborhood of v, N_2, such that for all $x \in N_2$, $x \notin \chi$. Then, F puts positive weight on $N = N_1 \cap N_2$ with $N \cap \chi = \emptyset$, which is a contradiction. Thus, 2 is necessary.

Sufficiency. Assume F satisfies BC and 2 for ψ. Because it satisfies 2, it clearly maximizes $L(G, \psi) = \mathbb{E}_G[U(\theta) - kH(\theta) - \psi\theta]$. Define the auxiliary Lagrangian $\tilde{L}$ as $\tilde{L}(G, \lambda) = \mathbb{E}_G[U(\theta) - kH(\theta) - \lambda \cdot (1, -1)\theta]$, for $\lambda \in \mathbb{R}^2$.

Because F maximizes L, it must also maximize $\tilde{L}$ when $\lambda \cdot (1, -1) = \psi$. Take $\lambda > 0$ such that this is the case which is, of course, always possible. Luenberger (1997), Chapter 8.4, Theorem 1 shows that if F maximizes $\tilde{L}$ it also solves:

$$\begin{aligned} \max_{G\in\Delta(\Theta)} \quad & \mathbb{E}_G[U(\theta) - kH(\theta)] \\ \text{s.t.} \quad & \mathbb{E}_G[\theta] \leq \mathbb{E}_F[\theta] \\ & -\mathbb{E}_G[\theta] \leq -\mathbb{E}_F[\theta] \end{aligned}$$

Because F satisfies BC, $\mathbb{E}_F[\theta] = \mu$. Thus, the problem above is equivalent to the acquisition problem. ■

Proof of Lemma 1

By the characterization of IC, U is differentiable almost everywhere, except at discontinuities of q and $U'(\theta) = q(\theta)$. If q is continuous at $\theta \in \operatorname{supp} F \cap (\underline{\theta}, \overline{\theta})$, then first order condition is necessary and implies:

$$q(\theta) - kH'(\theta) = \psi$$

We want to prove that, indeed, q is continuous in $\operatorname{supp} F \cap (\underline{\theta}, \overline{\theta})$, so the equality above holds. Assume, to obtain a contradiction, that this is not the case, so there is θ in that set such that q is discontinuous. By IC, q is monotonic, so we must have:

$$\lim_{z\uparrow\theta} q(z) < \lim_{z\downarrow\theta} q(z)$$

We start proving that θ is not a discontinuity point of F.

F **is not discontinuous at** θ**.** Assume that is the case. Then, pick a small $\varepsilon > 0$. Denote $\theta_+ \equiv \theta + \varepsilon$ and $\theta_- \equiv \theta - \varepsilon$. Define $\tilde{F}$ such that:

$$\tilde{F}(v) = \begin{cases} F(v) & \text{, if } v < \theta_- \\ F(v) + \frac{dF(\theta)}{2}) & \text{, if } v \in [\theta_-, \theta) \\ F(v) - \frac{dF(\theta)}{2}) & \text{, if } v \in [\theta, \theta_+) \\ F(v) & \text{, if } v \geq \theta_+ \end{cases}$$

$\tilde{F}$ clearly satisfies Bayesian consistency. Now consider:

$$\begin{aligned} &\mathbb{E}_{\tilde{F}}[U - kH] - \mathbb{E}_F[U - kH] = \\ \Big(U(\theta_-) - H(\theta_-)\Big)\frac{dF(\theta)}{2} + \Big(U(\theta_+) - kH(\theta_+)\Big)&\frac{dF(\theta)}{2} - \Big(U(\theta) - kH(\theta)\Big)dF(\theta) \\ &= \left(\int_\theta^{\theta_+} (q(v) - kH'(v))dv - \int_{\theta_-}^{\theta} (q(v) - kH'(v)dv\right)\frac{dF(\theta)}{2} \\ &\geq \left(\lim_{v\downarrow\theta} q(v) - \lim_{v\uparrow\theta} q(v)\right)\varepsilon\frac{dF(\theta)}{2} + k(H'(\theta_-) - H'(\theta_+))\varepsilon\frac{dF(\theta)}{2} \geq 0 \end{aligned}$$

where the last inequality holds for small enough ε, as the term in the first parentheses is bounded away from zero, whereas the term in the second parentheses goes to zero as ε approaches zero. This implies that $\tilde{F}$ increases the value of the objective function of the agent, which is a contradiction with optimality of F, so θ cannot be a point of discontinuity of F.

F **is not continuous at** θ**.** If that was the case, there would be a neighborhood N of θ such that $N \subset \operatorname{supp} F$. Let $\varepsilon > 0$ and $\theta_- = \theta - \varepsilon$, $\theta_+ = \theta + \varepsilon$, such that $\theta_-, \theta_+ \in N$. Notice that ε can be taken so that q is continuous at both θ_- and θ_+ - as q is increasing, it has at most countable discontinuities. Continuity of q at $\theta_-, \theta_+ \in \operatorname{supp} F$ implies, as shown before:

$$q(\theta_+) - kH'(\theta_+) = \psi = q(\theta_-) - kH'(\theta_-)$$

which can be reorganized as:

$$q(\theta_+) - q(\theta_-) = kH'(\theta_+) - kH'(\theta_-)$$

By taking ε sufficiently small, the right hand side can be made as close to zero as one desires - as H' is continuous, whereas the right hand side is bounded below by $\lim_{v\downarrow\theta} q(v) - \lim_{v\uparrow\theta} q(v)$, providing the desired contradiction.

As a consequence of the last two paragraphs, we obtained a contradiction with q discontinuous in $\operatorname{supp} F \cap (\underline{\theta}, \overline{\theta})$. Then, and if θ is in that set, $q(\theta) - H'(\theta) = \psi$. We finish the proof by showing the result for $\overline{\theta} \in \operatorname{supp} F$. The result for $\underline{\theta}$ is symmetric.

If $\overline{\theta} \in \operatorname{supp} F$. Assume $q(\overline{\theta}) - kH'(\overline{\theta}) < \psi$. Consider $\varepsilon > 0$:

$$\begin{aligned} U(\overline{\theta}) - kH(\overline{\theta}) - \psi\overline{\theta} - (U(\overline{\theta}-\varepsilon) - kH(\overline{\theta}-\varepsilon) - \psi(\overline{\theta}-\varepsilon)) = \\ \int_{\overline{\theta}-\varepsilon}^{\overline{\theta}} (q(z) - kH'(z))dz - \psi\varepsilon \\ \leq (q(\overline{\theta}) - KH'(\overline{\theta}-\epsilon) - \psi)\varepsilon \end{aligned}$$

Where the last inequality comes from monotonicty of q and convexity of H. By continuity of H' and the assumption that $q(\overline{\theta}) - H'(\overline{\theta}) < \psi$, for sufficiently small ε the last expression must become smaller than zero, finding a contradiction with 2. Therefore, $q(\overline{\theta}) - H'(\overline{\theta}) \geq \psi$. A similar argument establishes the result for $\underline{\theta}$, so necessity of M is concluded. ■

Proof of Lemma 2

Throughout this proof, we say that a menu is feasible for the principal if it satisfies IC and IA. Notice that 2 implies:

$$U(\theta) - kH(\theta) - \psi\theta \geq -kH(\theta_o(\psi)) - \psi\theta_o(\psi) \quad \text{for all} \quad \theta \in \operatorname{supp} F \tag{3}$$

Fix F and let a feasible (U,q) and ψ satisfy 3 with strict inequality. Consider the alternative menu $(\tilde{U},\tilde{q})$ with $\tilde{U}(\theta) = \max\{U(\theta) - \varepsilon, 0\}$, for some $\varepsilon > 0$ such that $(\tilde{U},q)$ still satisfies 3, and $\tilde{q}(\theta) = q(\theta)\mathbb{1}_{\tilde{U}(\theta)>0}$. Such an ε exists, because we assumed 3 held with strict inequality. Next, we show this menu is feasible. We finish the proof by showing that it is also more profitable.

Feasibility. First, it satisfies IA. To see that, recall by Lemma 3 that IA is equivalent to 2. Then, because (U,q) is feasible, 2 holds for it:

$$\operatorname{supp} F \subseteq \arg\max_{v\in\Theta}\{U(v) - kH(v) - \psi v\}$$

However, $\tilde{U}(v) - kH(v) - \psi v = \max\{U(v) - kH(v) - \psi v - \epsilon, -kH(v) - \psi v\}$, which is a monotonic transformation of $U(v) - kH(v) - \psi v$. Thus:

$$\arg\max_{v\in\Theta}\{U(v) - kH(v) - \psi v\} = \arg\max_{v\in\Theta}\{\tilde{U}(v) - kH(v) - \psi v\}$$

and IA holds for the new menu $(\tilde{U},q)$.

For IC, notice that $\tilde{q}$ is increasing and satisfies individual rationality by definition. Finally:

$$\tilde{U}(\theta) = \max\{0, U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} q(v)dv - \varepsilon\} = \max\{0, U(\underline{\theta}) - \varepsilon\} + \int_{\underline{\theta}}^{\theta} q(v)\mathbb{1}_{\tilde{U}(v)>0}dv$$

so the envelope condition also holds and $(\tilde{U}, \tilde{q})$ satisfy IC. Thus, it is feasible.

Profitability. We finally show $(\tilde{U}, \tilde{q})$ is more profitable than (U, q). Notice that $\tilde{U} < U$. We prove that, in the support of F, $q = \tilde{q}$. To see that, take $\theta \in \operatorname{supp} F$. Because $(\tilde{U}, \tilde{q})$ is feasible, we have, by reordering 3:

$$\begin{aligned}\tilde{U}(\theta) &\geq kH(\theta) - kH(\theta_o(\psi)) + \psi(\theta - \theta_o(\psi)) \\ &\geq kH(\theta) - kH(\theta_o(\psi)) - kH'(\theta_o(\psi))(\theta - \theta_o(\psi)) > 0\end{aligned}$$

where the first inequality comes from reordering 3, the second from the definition of $\theta_o(\psi)$ and the third from strict convexity of H. Then, $\tilde{U}(\theta) > 0$, implying $\tilde{q}(\theta) = q(\theta)$. Because $\tilde{U} < U$ and, in the support of F, $\tilde{q} = q$, $(\tilde{U}, \tilde{q})$ must be more profitable than (U, q). Thus, (U, q) cannot be optimal, because $(\tilde{U}, \tilde{q})$ is also feasible. Therefore, for any optimal menu, 3 must hold with equality: that is, TP holds, as we wanted to prove. ■

Lemma 4 (Binary is Sufficient). *Assume the principal's problem has a solution. Then, there are $F, (U, q)$ solving it such that $|\operatorname{supp} F| \leq 2$ and $|q(\Theta)/\{0\}| \leq 2$.*

Proof of Lemma 4

By Lemma 3, the problem of the principal can be written as:

$$\max_{G,(U,q),\psi} \{\mathbb{E}_G[\theta q(\theta) - c(q(\theta)) - U(\theta)] : IC,\ BC \quad \text{and} \quad 2\}$$

where we have rewritten profits in the usual surplus minus rents format. Fix any solution to this problem, $(U, q), F, \psi$. First, we show it is sufficient to focus on binary distributions. Then we show we can restrict the menu accordingly.

Binary distribution. For fixed $(U, q), \psi$, we show the principal is at least as well off with a binary distribution. Consider the simplex $\Delta(\operatorname{supp} F)$. By Winkler (1988), the problem of maximizing profits in this simplex subject to BC is solved by an at-most binary distribution, call it $\tilde{F}$. Because $\tilde{F}$ solves this problem, it is at least as profitable as F for the principal. We now show it is feasible under the principal's problem.

Notice that IC does not depend on F, so $(U, q), \psi, \tilde{F}$ satisfy IC. Additionally, because $\operatorname{supp} \tilde{F} \subset \operatorname{supp} F$, then $(U, q), \psi, \tilde{F}$ satisfy 2. Thus, $\tilde{F}$ is feasible. That implies that focusing on at-most-binary distributions is sufficient.

Binary menus. Now start with a solution $(U,q),\psi,F$ with F at-most-binary. Define the following alternative menu:

$$\tilde{U}(\theta)=\max\left\{0,\max_{v\in\operatorname{supp}F}\{U(v)+(\theta-v)q(v)\}\right\}$$

Let $v(\theta)\in\operatorname{supp}F$ be the largest argmax of the maximization problem in the parentheses above. We define $\tilde{q}(\theta)=q(v(\theta))\mathbb{1}_{\tilde{U}(\theta)>0}$. Notice that $(\tilde{U},\tilde{q})$ was constructed to satisfy IC. It is also easy to see it coincides with (U,q) in $\operatorname{supp}F$, so it satisfies 2. Now, this new menu produces exactly the same profits as (U,q), but notice that $q(\Theta)\subset q(\operatorname{supp}F)\cup\{0\}$, proving the result, since $\operatorname{supp}F$ is at-most-binary. ■

Proof of Proposition 1

Recall the problem of the principal can be written as:

$$\max_{G,(U,q)}\{\mathbb{E}_G[\theta q(\theta)-c(q(\theta))-U(\theta)]:IC\quad\text{and}\quad IA\}\tag{4}$$

where we have rewritten profits in the usual surplus minus rents format. We call the value of this problem L^*. Let the value of problem P be L^P. We start showing that $L^*\leq L^P$. Then, we prove that the converse holds by constructing a transformation that allows us to express the solution to one of the problems as a solution to the other.

First, $L^*\leq L^P$. Take a solution of 4, $\{F,(U,q)\}$. By Lemma 4, assume without loss of generality that F is at-most-binary and $|q(\Theta)/\{0\}|\leq 2$. Because the solution must satisfy IA, by Lemma 3, it also satisfies 2 for some ψ, and F is Bayesian consistent. Additionally, by Lemma 1, IA and IC imply M, and by Lemma 2, optimality in the principal's problem 4 implies TP.

Just as in the text, identify $\operatorname{supp}F=\{\theta_L,\theta_H\}$, with $\theta_L\leq\mu\leq\theta_H$, and then define $q_i\equiv q(\theta_i)$ and $U_i\equiv U(\theta_i)$, $i\in\{L,H\}$. Then, $\left\{F,\{U_i,q_i\}_{i\in\{L,H\}},\psi\right\}$ is feasible in P, because TP and M depend only on the points in the support of F. Additionally, it is easy to see that profits under $\left\{F,\{U_i,q_i\}_{i\in\{L,H\}},\psi\right\}$ in P are the same as L^*. Because L^P is the maximum profits obtained at P, $L^*\leq L^P$.

Next, we prove that $L^P\leq L^*$. Take a feasible element of P, $\left\{F,\{U_i,q_i\}_{i\in\{L,H\}},\psi\right\}$. We show we can construct from that a feasible element of 4, $\{F,(U,q)\}$ that preserves the profits of the principal. The proof proceeds in two steps.

Step 1: Extension to a menu that satisfies IC. For $\theta_i\in\operatorname{supp}F$, Let $q(\theta_i)\equiv q_i$ and $U(\theta_i)\equiv U_i$, so these functions are defined in $\operatorname{supp}F$. We now extend them. Define, for $\theta\notin\operatorname{supp}F$:

$$U(\theta) = \max\{0, \max_{v \in \operatorname{supp} F}\{U(v) + (\theta - v)q(v)\}\}$$

Let $v(\theta)$ be the largest argmax of the maximization problem in the parentheses above. We define $\tilde{q}(\theta) = q(v(\theta))\mathbb{1}_{\tilde{U}(\theta)>0}$. (U, q) is clearly individually rational. We prove that is it also incentive compatible. Start with $\theta, \theta' \in \operatorname{supp} F$. We have:

$$U(\theta) - U(\theta') - (\theta - \theta')q(\theta') =$$

$$U(\theta') + kH(\theta) - kH(\theta') + \psi(\theta - \theta') - U(\theta') - (\theta - \theta')q(\theta') \geq$$

$$kH(\theta) - kH(\theta') + \psi(\theta - \theta') - (\theta - \theta')(\psi + kH'(\theta')) \geq 0$$

where the first equality is a consequence of TP, the first inequality cancels repeated terms and applies M, and the second inequality cancel the new repeated terms and uses convexity of H. This implies incentive compatibility holds in $\operatorname{supp} F$. It is easy to see that U was constructed so that it satisfied incentive compatibility outside of the support, so (U, q) satisfy IC, finishing step 1.

Step 2. $F, (U, q)$ **satisfy IA** We start by proving that $g(\theta) \equiv U(\theta) - kH(\theta) - \psi\theta$ is concave by parts. Notice that, $q(\Theta) \subset \{q_o \equiv 0, q_L, q_H\}$. q is non-decreasing, because (U, q) satisfies IC, so we can define the intervals $I_i = \{\theta \in \Theta : q(\theta) = q_i\}$, for $i \in \{o, L, H\}$. Now, $g|_{I_i}$ is differentiable and we have:

$$g'|_{I_i}(\theta) = q_i - H'(\theta) - \psi$$

which is decreasing, so $g|_{I_i}$ is concave for each $i \in \{o, L, H\}$. Then, by M and the definition of $\theta_o \equiv \theta_o(\psi)$, for each i, $\theta_i \in \arg\max_{v \in I_i}\{g(v)\}$. That implies:

$$\arg\max_{v \in \Theta}\{g(v)\} \subseteq \max_{i \in \{o, L, H\}}\{g(\theta_i)\}$$

But by TP, $g(\theta_i)$ is a constant across i's. Because $\operatorname{supp} F = \{\theta_L, \theta_H\}$ we have:

$$\operatorname{supp} F \subseteq \arg\max_{v \in \Theta} g(\theta)$$

which is exactly 2 which, by Lemma 3 implies IA, when summed with the fact that F satisfies BC.

We proved that $\{F, (U, q)\}$ is feasible under 4. But because the menu coincides with $\{U_i, q_i\}_{i \in \{L, H\}}$ in the support of F, it obtains the same profit in 4 as in P. Because this was done for an arbitrary feasible element of P we have: $L^P \leq L^*$.

We have then established $L^P = L^*$ and constructed a mapping between solutions to the problems, proving that they are equivalent.

Existence of Solution. As previously argued, P is an optimization in 7 variables. Recall that we identify F with its support: $\{\theta_L, \theta_H\}$, $\theta_L \leq \mu \leq \theta_H$. Notice that $p_L^F = 1 - p_H^F$, and $p_H^F = \frac{\mu - \theta_L}{\theta_H - \theta_L}$, for $\theta_H > \theta_L$. Given M, For $\theta_H = \theta_L = \mu$, we have $q_H = q_L$ and $U_H = U_L$. Because of that, p_H^F is immaterial in that case, so we define $p_H^F = 0$. By inspection, the restriction of the objective function to the constraint set is continuous in $\left\{\{\theta_i, U_i, q_i\}_{i \in \{L,H\}}, \psi\right\}$. We proceed to prove the constraint set is compact.

M, TP and BC are clearly closed. As previously argued, q_i, U_i are bounded, and $\theta_i \in \Theta$ are also bounded, for $i \in \{L, H\}$. Then, we just need to prove that ψ is bounded. By M, we have, for $\theta_L < \theta_H$:

$$q_H - q_L \geq kH'(\theta_H) - kH'(\theta_L)$$

Notice that this implies $H'(\theta_i)$, $i \in \{L, H\}$ is uniformly bounded. To see that, assume there is a sequence of feasible choices, with θ_i^n, and $H'(\theta_H^n) \geq n$ unbounded. Then, because q_i are bounded, we have that $H'(\theta_L^n)$ also grows unboundedly, so that the right hand side of the inequality above remains bounded. But this is impossible because $\theta_L^n \leq \mu$ implies $kH'(\theta_L^n) \leq kH'(\mu) < \infty$. A similar argument holds if we assume θ_L^n is unbounded below. Then, using M again we obtain:

$$q_L - kH'(\theta_L) \leq \psi \leq q_H - kH'(\theta_H)$$

which guarantees that ψ is bounded. Thus, P is a problem of maximizing a continuous function over a compact set and, therefore, it has a solution. ■

Proof of Proposition 2

We start by constructing a Lagrangian for the principal's problem. For that, we add two auxiliary variables to the problem. Write

$$q_i = kH'(\theta_i) + \psi + a_i$$

By M, $a_L \leq 0$, with equality for all $\theta_L > \underline{\theta}$ and $a_H \geq 0$, with equality for all $\theta_H < \overline{\theta}$. With that definition, we can eliminate M and plug q in the profit function. Additionally, recall that $\theta_o(\psi) = \arg\max_{v \in \Theta}\{-kH(v) - \psi v\} \leq \theta_L$, as $q_L \geq 0$. Using that, we solve TP for U_i to obtain:

$$U_i = kH(\theta_i) + \psi\theta + \max_{v\in\Theta}\{-kH(v) - \psi v\}$$

This then allows us to eliminate the constraint TP and to, again, rewrite the profit function plugging this equation for U_i. In order for the problems to be equivalent, then, we need to impose two types of constraints on a_i. In particular, $a_L \leq 0$, $a_H \geq 0$, to which we associate multipliers α_i, and $a_L(\theta_L - \underline{\theta}) = 0$ and $a_H(\overline{\theta} - \theta_H) = 0$, to which we associate β_i. For shortness, we encode $\vartheta_L = \underline{\theta}$ and $\vartheta_H = \overline{\theta}$. Finally, recall that $q_i \geq 0$ is also a constraint, that we associate with multiplier y_i. We can then write the following Lagrangian for P, plugging in the equation above for U_i:

$$\begin{aligned}\mathcal{L}(\{\theta_i, a_i, \alpha_i, \beta_i, y_i\}_{i\in\{L,H\}}, \psi) = \sum_{i\in\{L,H\}} p_i^F \Big\{ & \theta_i\left(kH'(\theta_i) + \psi + a_i\right) - c\left(kH'(\theta_i) + \psi + a_i\right) \\ & - \Big(kH(\theta_i) + \psi\theta_i + \max_{v\in\Theta}\{-kH(v) - \psi v\}\Big) \\ & - \alpha_i a_i - \beta_i a_i(\theta_i - \vartheta_i) - y_i\left(kH'(\theta_i) + \psi + a_i\right)\Big\}\end{aligned} \tag{5}$$

Notice that, by M, $q_H \geq q_L$, so $y_H = 0$, and we omit it whenever convenient. Start with first order conditions for ψ:

$$[\psi]: \quad \mu - \theta_o(\psi) = \sum_{i\in\{L,H\}} p_i^F[\theta_i - c'(q_i) - y_i] \tag{6}$$

Then, for a_i:

$$[a_L]: \quad \theta_L - c'(q_L) \geq \alpha_L + \beta_L(\theta_L - \underline{\theta}) + y_L \tag{7}$$

with equality if $a_L < 0$, and:

$$[a_H]: \quad \theta_H - c'(q_H) \leq \alpha_H + \beta_H(\theta_H - \overline{\theta}) \tag{8}$$

with equality if $a_H > 0$.

We prove, from now, that q_L is underprovided.

q_L **is underprovided — $a_L = 0$.** If $q_L = 0$, this is obvious. So we prove it for $q_L > 0$ — thus, $y_L = 0$. First, if $\theta_L < \theta_H$, equation 6 and strict convexity of c imply:

$$c'(q_L) < \sum_i p_i^F c'(q_i) = \theta_o(\psi) \leq \theta_L$$

By contrast, if $\theta_L = \mu$, by 6:

$$c'(kH'(\mu) + \psi) = \theta_o(\psi)$$

By a simple application of Topkis' lemma, $\theta_o(\psi)$ is nonincreasing with ψ. Define: $g(\psi) = c'(kH'(\mu) + \psi) - \theta_o(\psi)$, which is then strictly increasing with ψ. Additionally, $g(-kH'(\mu)) = -\mu < 0$, and g is unbounded — because $\theta_o(\psi) \leq \theta_L$. Thus, there is a unique $\psi > -kH'(\mu)$ with $g(\psi) = 0$. Because $\psi > -kH'(\mu)$, $\theta_o(\psi) < \mu$ and we have:

$$c'(q) = \theta_o(\psi) < \mu$$

as we wanted to prove. We finish by arguing that this implies $a_L = 0$. Assume $a_L < 0$. Because we just proved that q_L is underprovided, we can increase a_L, which would increase q_L, improving profits, while not violating any constraint. Then, it must be that $a_L = 0$.

First Order Conditions for θ_i. Now, define $L_i \equiv \theta_i q_i - c(q_i) - (kH(\theta_i) + \psi\theta + \max_{v \in \Theta}\{-kH(v) - \psi v\})$. Also, let $p^F \equiv p^F_H$. For $\theta_H > \theta_L$:

$$\frac{dp^F}{d\theta_H} = -\frac{p^F}{\theta_H - \theta_L} \qquad \text{and} \qquad \frac{dp^F}{d\theta_L} = -\frac{(1 - p^F)}{\theta_H - \theta_L}$$

We start by focusing on the case in which $\theta_i \neq \mu$, $i \in \{L, H\}$, so $\theta_H > \theta_L$. Take first order conditions of $\mathcal{L}$ with respect to θ_i to obtain:

$$[\theta_H]: \qquad -\frac{p^F}{\theta_H - \theta_L}(L_H - L_L) + p^F\left(a_H + (\theta_H - c'(q_H))\, kH''(\theta_H)\right) - p^F \beta_H a_H \geq 0 \tag{9}$$

with equality if $\theta_H < \overline{\theta}$, and:

$$[\theta_L]: \qquad -\frac{(1 - p^F)}{\theta_H - \theta_L}(L_H - L_L) + (1 - p^F)(\theta_L - c'(q_L) - y_L)\, kH''(\theta_L) \leq 0 \tag{10}$$

with equality if $\theta_L > \underline{\theta}$.

We now prove the main results of the proposition.

High Quality is underprovided. From the first order conditions for θ_i:

$$(1 - \beta_H)a_H + (\theta_H - c'(q_H))\, kH''(\theta_H) \geq (\theta_L - c'(q_L) - y_L)\, kH''(\theta_L) \tag{11}$$

with equality if $\underline{\theta} < \theta_L < \theta_H < \overline{\theta}$.

We proceed by analysing two cases. Start with $a_H = 0$. Then, the inequality above becomes:

$$(\theta_L - c'(q_L) - y_L)kH''(\theta_L) \leq (\theta_H - c'(q_H))kH''(\theta_H)$$

Because the left hand side of 6 is positive and H is strictly convex, we have that at least one of the sides of inequality above is positive. Thus $c'(q_H) < \theta_H$.

Now, assume $a_H > 0$. In this case, notice that it must be $\theta_H = \overline{\theta}$ and $\alpha_H = 0$. Then, by 8, $\theta_H = c'(q_H)$, and we are done.

Aggregate distortions. We can rewrite 6 as:

$$\sum_{i\in\{L,H\}} p_i^F c'(q_i) = \theta_o(\psi) - p_L^F y_L \leq \max\{\theta_L, p_H^F \theta_H\} = \sum_{i\in\{L,H\}} p_i^F c'(q_i^s)$$

where the first inequality comes from $q_L \geq 0$ and the definition of $\theta_o(\psi)$, and the last equality by the known pure screening solution. To see that the inequality in the middle holds, consider the following two cases. First, if $q_L > 0$, $y_L = 0$ and the inequality is true by $\theta_o(\psi) \leq \theta_L$. Now, if $y_L \neq 0$, we have, by 6:

$$\tau(F, q) = p_H^F c'(q_H) \leq p_H^F \theta_H$$

using the result that q_H is underprovided. That proves the aggregate distortions result. We next prove these two results when $\theta_i = \mu$ for some i.

No Acquisition: $\operatorname{supp} F = \{\mu\}$**.** We proved before that:

$$c'(q) = \theta_o(\psi) < \mu$$

This shows that both the underprovision and the aggregate distortion results hold strictly in this case.

k large enough. Recall that, q_i, $i \in \{L,H\}$ is bounded — uniformly on k by strict convexity of c. Just as in the proof of existence in Proposition 1, this implies that $\{kH'(\theta_i)\}_{i\in\{L,H\}}$ is uniformly bounded on k. Thus, for large enough k, $kH'(\theta_H) < kH'(\overline{\theta})$, as the term in the right grows unbounded. Thus, because we proved that $c'(q_H) = \theta_H$ only when $\theta_H = \overline{\theta}$, we have that, for high enough k, $c'(q_H) < \theta_H$.

Notice that, with the same argument as above, we can conclude that, for high enough k, $\theta_L, \theta_H \in (\underline{\theta}, \overline{\theta})$. Recall that 6 can be written as

$$\sum_i p_i^F c'(kH'(\theta_i)+\psi) = \theta_o(\psi) - p_L^F y_L$$

If $q_L = 0$, because $\overline{\theta} > \theta_H$, $c'(q_H) < \theta_H$ so $\sum_i p_i^F c'(kH'(\theta_i)+\psi) < p_H^F \theta_H$. If, on the other hand, $q_L > 0$, $y_L = 0$ and because $\theta_L > \underline{\theta}$, we have $\theta_L > \theta_o(\psi)$. Applying both of these arguments to the equation above, we get:

$$\sum_i p_i^F c'(q_i) < \max\{\theta_L, p_H^F \theta_H\} = \sum_i p_i^F c'(q_i^s)$$

H is UMC. If that is the case, then for $k > 0$ it is clear that $\theta_i \in (\underline{\theta}, \overline{\theta})$ for $i \in \{L, H\}$. Thus the same argument holds as for when k is large enough. ■

Proof of Proposition 3

We use the same notation and refer to equations in the proof of Proposition 2. We use references to equations in the main appendix.

The proof is divided in several steps. We start by showing that, if $|\operatorname{supp} F| = 2$, then $\theta_H = \overline{\theta}$. When $|\operatorname{supp} F| = 1$, we know $\operatorname{supp} F = \{\mu\}$ so we abuse notation to say that $\theta_H = \overline{\theta}$ always — implicitly setting $\theta_L = \mu$ when no information is acquired.

A general observation to be used in this proof is that M always binds for the low-type. That is, $q_L = kH'(\theta_L)+\psi$. To see that, assume that this is not the case, so $q_L < kH'(\theta_L)+\psi$. We proved that in equilibrium $c'(q_L) < \theta_L$. Thus, increasing q_L increases surplus while still satisfying M. Thus, M must hold with equality.

Step 1: $\theta_H = \overline{\theta}$. For a contradiction, assume that $\mu < \theta_H < \overline{\theta}$, which then implies $q_H = kH'(\theta_H) + \psi = k(\theta_H - \mu) + \psi$. By using the equation for the FOC of θ_H, 9, with $a_H = 0$, and inequality for the FOC of θ_L, 10, we obtain:

$$\theta_L - k(\theta_L - \mu) - \psi \leq \theta_H - k(\theta_H - \mu) - \psi$$

with equality if $\theta_L > \underline{\theta}$. This can be rewritten to obtain $(1-k)(\theta_H - \theta_L) \geq 0$. This can only hold with $k \leq 1$, and with equality only if for $k = 1$. That means $\theta_H = \overline{\theta}$ for $k > 1$. We proceed by proving that $\theta_H = \overline{\theta}$ for $k < 1$ and Berge's maximum theorem implies the result also holds for $k = 1$.

So assume $k < 1$. In that case, the inequality must hold and, therefore, $\theta_L = \underline{\theta}$. As a consequence, $\theta_o(\psi) = \underline{\theta}$ as well. We can then apply that to equation 6 to solve for ψ and get $\psi = \underline{\theta}$.

Finally, we can use this information to solve for θ_H in equation 9. We can calculate:

$$L_H - L_L = k(\theta_H - \underline{\theta})(\theta_H - \mu) - \frac{1+k}{2}k\left((\theta_H - \mu)^2 - (\underline{\theta} - \mu)^2\right)$$

Applying that to equation 9 generates, after simplification, the equality

$$\frac{1-k}{2}(\theta_H - \underline{\theta})^2 = 0$$

which cannot hold for $k \neq 1$. Proving that $\theta_H = \overline{\theta}$.

Step 2: $q_H = \overline{\theta}$. This argument can be made generically when $\theta_H = \overline{\theta}$. To see that, recall that Proposition 2 shows that $c'(q_H) \leq \theta_H$. Now, assume $\theta_H = \overline{\theta}$ and $c'(q_H) < \overline{\theta}$ with $q_H \geq kH'(\theta_H) + \psi$. Then increasing q_H increases the surplus but maintains M, implying that at the optimum $q_H = \overline{\theta}$.

In the next steps we look for solutions in each possible scenario:

a. $\underline{\theta} < \theta_o < \theta_L$

b. $\underline{\theta} < \theta_o = \theta_L$

c. $\underline{\theta} = \theta_o$

For that, let p be the probability of the high type under F. By Bayesian Consistence, $p = \frac{\theta_L - \mu}{\overline{\theta} - \theta_L}$. Denote by π the probability of high type under the prior — equivalently, under full information.

Step 3: Scenario a — $\underline{\theta} < \theta_o < \theta_L$. We start assuming $\theta_L < \mu$. By definition of the threat point and the assumption $\theta_o > \underline{\theta}$, we have:

$$k(\mu - \theta_o) = \psi$$

and, thus, $q_L = k(\theta_L - \theta_o) > 0$, implying $y_L = 0$. From equation 6, we can solve for θ_o using the expression for p above to obtain:

$$\theta_o = \frac{\theta_L((k-1)\overline{\theta} - k\mu) + \mu\overline{\theta}}{(k+1)\overline{\theta} - k\mu - \theta_L} \tag{12}$$

Note that now we have:

$$L_H - L_L = \frac{\overline{\theta}^2}{2} - \frac{k(\overline{\theta} - \mu)^2}{2} - k(\mu - \theta_o)\overline{\theta} - \theta_L k(\theta_L - \mu) + k^2\frac{(\theta_L - \theta_o)^2}{2} + k\frac{(\theta_L - \mu)^2}{2}$$

According to the FOC of θ_L in equation 10 it must hold that:

$$L_H - L_L = k(\theta_L - k(\theta_L - \theta_o))(\overline{\theta} - \theta_L)$$

Solving this equation for θ_o we obtain only one positive solution:

$$\theta_o = \beta(k)\Big((1+\beta(k))\overline{\theta} - \theta_L\Big) \tag{13}$$

with $\beta(k) = \sqrt{\frac{k-1}{k}}$ Together with equation 12, the equation above determines θ_o and θ_L. We can solve them together to obtain the explicit formulas:

$$\theta_L(k) = \beta(k)^{-1}\Big((\beta(k) - k^{-1})\overline{\theta} + (\beta(k)+1)k(\overline{\theta} - \mu)\Big) \tag{14}$$

$$\theta_o(k) = \overline{\theta} - (\beta(k)+1)k(\overline{\theta} - \mu)$$

We have then found the solution to the problem for this case. For this solution to hold, there are three necessary conditions: $\underline{\theta} < \theta_o$, $\theta_o < \theta_L$ and $\theta_L \le \mu$. Start by noticing that the expression for θ_L is increasing in k and the expression for θ_o is decreasing in k. Then, there is a (possibly empty) interval such that the three conditions above hold. First, define k_0 such that $\theta_L(k_0) = \theta_o(k_0)$. That is:

$$k_0 = 1 + \frac{\left(\sqrt{\frac{\overline{\theta}}{\overline{\theta}-\mu}} - 1\right)^2}{2\sqrt{\frac{\overline{\theta}}{\overline{\theta}-\mu}} - 1}$$

Importantly, $k_0 > 1$. Now, define k_2 by $\theta_L(k_2) = \mu$. It is easy to see that there is exactly one k_2 solving that. Additionally, $k_0 < k_2$. To see that, note that, by equation 13, $\theta_L(k_0) = \beta(k_0)\overline{\theta}$. Then, $\theta_L(k_0) \le \mu$ if and only if $k_0 \le \frac{\overline{\theta}^2}{\overline{\theta}^2 - \mu^2}$, which is always true given our formula for k_0.

Finally, let k_1 be such that $\theta_o(k_1) = \underline{\theta}$. Then:

$$k_1 = \frac{1}{1-\pi^2}$$

where π is the probability of $\overline{\theta}$ under the prior. We have $k_1 \ge k_0$ if and only if $\pi \ge \frac{\underline{\theta}}{\overline{\theta}}$. And, finally, because θ_L is increasing, we can solve the inequality $\theta_L(k_1) \ge \mu$ to find out that:

$$k_1 \ge k_2 \iff \pi \ge \hat{\pi} \equiv \frac{\underline{\theta} - \overline{\theta} + \sqrt{(\overline{\theta} - \underline{\theta})^2 + 4\underline{\theta}^2}}{2\underline{\theta}}$$

We now find the result for Scenario a when $\theta_L = \mu$. In that case, equation 6 implies:

$$\theta_o(k) = \frac{k}{k+1}\mu$$

We define k_3 such that $\theta_o(k_3) = \underline{\theta}$. With that $k_3 = \frac{\underline{\theta}}{\overline{\theta}-\underline{\theta}}\frac{1}{\pi}$. It is easy to see that this subcase kicks in for $k \geq k_2$, when the solution from the previous paragraphs would prescribe $\theta_L > \mu$. To summarize our results in this section, we have:

θ_L is given by equation 14 for $k \in [k_0, \min\{k_1, k_2\}]$ when $\pi \geq \frac{\underline{\theta}}{\overline{\theta}}$. $\theta_L = \mu$ for $k \geq \max\{k_2, k_3\}$. And $\hat{\pi}$ determines which is larger between k_1 and k_2.

Step 4: Scenario b — $\underline{\theta} < \theta_o = \theta_L$. In this case, we must have $q_L = 0$. Using equation 6, we can solve for y_L to obtain $y_L = \frac{\theta_L(\overline{\theta}-\theta_L)-(\mu-\theta_L)\overline{\theta}}{\overline{\theta}-\mu}$. Again, from the definition of θ_o, and the interiority assumption, $\psi = k(\mu - \theta_L)$, using the fact that $\theta_L = \theta_o$.

Notice by equation 10, we must have:

$$L_H - L_L = k(\theta_L - y_L)(\overline{\theta} - \theta_L)$$

In this case, L_H is the same as in Scenario a, but L_L changes so:

$$L_H - L_L = \frac{\overline{\theta}^2}{2} - \frac{k(\overline{\theta}-\mu)^2}{2} - k(\mu-\theta_L)\overline{\theta} + k\frac{(\theta_L-\mu)^2}{2} + k(\mu-\theta_L)\theta_L$$

We can then solve for θ_L obtaining one real solution:

$$\theta_L(k) = \frac{1}{6}\left(-\frac{k(\mu-\overline{\theta})^2}{\sqrt[3]{54k^2\overline{\theta}^2(\overline{\theta}-\mu) + 6\sqrt{3}\sqrt{k^4\overline{\theta}^2(\mu-\overline{\theta})^2\left(k(\mu-\overline{\theta})^2 + 27\overline{\theta}^2\right) - k^3(\mu-\overline{\theta})^3}}} - \frac{\sqrt[3]{54k^2\overline{\theta}^2(\overline{\theta}-\mu) + 6\sqrt{3}\sqrt{k^4\overline{\theta}^2(\mu-\overline{\theta})^2\left(k(\mu-\overline{\theta})^2 + 27\overline{\theta}^2\right) - k^3(\mu-\overline{\theta})^3}}}{k} + \mu + 5\overline{\theta}\right) \tag{15}$$

Differentiation guarantees that this expression is increasing for k. We can then define $\hat{k}$ such that $\theta_L(k) = \underline{\theta}$. That generates:

$$\hat{k} = \frac{\overline{\theta}^2}{(\overline{\theta}-\underline{\theta})^2}\frac{1-\pi}{1+\pi}$$

Notice that $\hat{k} \leq k_0$ if and only if $\pi \geq \frac{\underline{\theta}}{\overline{\theta}}$. Similarly, simple algebra shows that $\hat{k} \geq k_1$ if and only if $\pi \geq \frac{\underline{\theta}}{\overline{\theta}}$. Thus, the solution to Scenario b satisfies equation 15 whenever $k \in [\hat{k}, k_0]$ and $\pi \geq \frac{\underline{\theta}}{\overline{\theta}}$.

Step 5: Scenario c — $\underline{\theta} = \theta_o$. In this case we have, by equation 6:

$$q_L + y_L = \frac{\underline{\theta}(\overline{\theta} - \theta_L) - (\mu - \theta_L)\overline{\theta}}{\overline{\theta} - \mu}$$

And $-k(\underline{\theta} - \mu) \leq \psi = q_L - k(\theta_L - \mu)$. With this we obtain that either $q_L > 0$ or $\theta_L = \underline{\theta}$. If $q_L > 0$:

$$L_H - L_L = \frac{\overline{\theta}^2}{2} - \frac{k(\overline{\theta} - \mu)^2}{2} - \left(\frac{\underline{\theta}(\overline{\theta} - \theta_L) - (\mu - \theta_L)\overline{\theta}}{\overline{\theta} - \mu} - k(\theta_L - \mu)\right)\overline{\theta} - \theta_L k(\theta_L - \mu) + \frac{\left(\frac{\underline{\theta}(\overline{\theta} - \theta_L) + (\mu - \theta_L)\overline{\theta}}{\overline{\theta} - \mu}\right)^2}{2} + k\frac{(\theta_L - \mu)^2}{2}$$

Using equation 9, we know:

$$L_H - L_L \geq k(\theta_L - \frac{\underline{\theta}(\overline{\theta} - \theta_L) - (\mu - \theta_L)\overline{\theta}}{\overline{\theta} - \mu})(\overline{\theta} - \theta_L)$$

We can then rewrite this inequality as:

$$\frac{(\overline{\theta} - \theta_L)^2\left(k(\mu - \overline{\theta})(\mu + \overline{\theta} - 2\underline{\theta}) + (\overline{\theta} - \underline{\theta})^2\right)}{2(\mu - \overline{\theta})^2} \geq 0$$

This inequality holds (strictly) for $k \leq (<) \frac{1}{1-\pi^2} = k_1$. By equation 10, this implies $\theta_L = \underline{\theta}$ for $k < k_1$. Continuity then implies $\theta_L = \underline{\theta}$ for $k \leq k_1$. We assumed $q_L > 0$, which means, in this case, $q_L = \underline{\theta} - \frac{\pi}{1-\pi}(\overline{\theta} - \underline{\theta}) > 0$ if and only if $\pi < \frac{\underline{\theta}}{\overline{\theta}}$.

The remaining case is when $q_L = 0$ — i.e. $\pi \geq \frac{\underline{\theta}}{\overline{\theta}}$. In that case, we know $\theta_L = \underline{\theta}$ and $\psi = -k(\underline{\theta} - \mu)$. With that:

$$L_H - L_L = \frac{\overline{\theta}^2}{2} - \frac{k(\overline{\theta} - \mu)^2}{2} - k(\mu - \theta_L)\overline{\theta} - \theta_L k(\theta_L - \mu) + k\frac{(\theta_L - \mu)^2}{2}$$

Using equation 10 again, we obtain: $k \leq \hat{k}$.

Step 6: General Solution. Because the optimality conditions are necessary and we have covered all possible parameters, we have a solution for the problem. Collecting all parts in the language of the proposition statement we can define $\omega(k)$ as follows: if $\pi \geq \frac{\underline{\theta}}{\overline{\theta}}$, $\omega(k) = \underline{\theta}$ for $k < \hat{k}$, it follows equation equation 15 for $k \in [\hat{k}, k_0)$, then it follows equation equation 14 for $k \in [k_0, \min\{k_1, k_2\}]$, after which $\omega(k) = \mu$.

In contrast, if $\pi < \frac{\underline{\theta}}{\overline{\theta}}$, then $\omega(k) = \underline{\theta}$ for $k \in [0, k_1]$ and μ otherwise. The optimal solution is then always equal to $\omega(k)$ and unique, except possibly at k_1 when any $\theta_L \in [\omega(k_1), \mu]$ is a solution to the problem. ■

Proof of Proposition 4

Let $Q(k)$ be the value function of P at k. We can apply the envelope theorem to 5 to conclude that the derivative of the profit function satisfies:

$$Q'(k) = -\sum_{i\in\{L,H\}} p_i^F \left\{H(\theta_i^k) - H(\theta_o(\psi^k)) - (\theta_i^k - c'(q_i^k) - y_i^k)H'(\theta_i^k)\right\} \tag{16}$$

where superscript k denotes that the variable solves P for k. We start the proof showing that profits are decreasing for small k and increasing for large k. Then we move on to consumer surplus.

Profits decrease for small k. When $k = 0$, the solution to P is full information and the optimal contract is the second-best contract for that information: $\operatorname{supp} F^0 = \operatorname{supp} \overline{F} = \{\underline{\theta}, \overline{\theta}\}$, $q^0 = q^s$. Notice that this implies $\theta_o(\psi^0) = \theta_L$. We know $c'(q_H^0) = \overline{\theta}$. Applying this in 6, we have

$$\theta_L - c'(q_L^0) - y_L = \frac{p_H^{\overline{F}}}{p_L^{\overline{F}}}(\overline{\theta} - \underline{\theta})$$

Plugging these into 16:

$$Q'(0) = -p_H^F\{H(\overline{\theta}) - H(\underline{\theta}) - H'(\underline{\theta})(\overline{\theta} - \underline{\theta})\} < 0$$

where the inequality is due to strong convexity of H. Then, we obtain the intended result.

Profits increase for large k. In Lemma 5, we prove that there is $\overline{k}$ such that, for $k \geq \overline{k}$ $\operatorname{supp} F^k = \{\mu\}$. It is direct that $\theta_o(\psi^k) < \mu$ for any finite k. Thus, using 6:

$$Q'(k) = -\{H(\mu) - H(\theta_o(\psi^k)) - H'(\mu)(\mu - \theta_o(\psi^k)\} > 0$$

for $k \geq \overline{k}$, where we used, again, strict convexity of H.

Consumer Surplus increases for small k. First, at $k = 0$ there is full information. If $q_L^s = 0$ for full information, we have that consumer surplus is zero at $k = 0$. Because consumer surplus is always positive at positive k, we have our result. By usual arguments, $q_L^s = 0$ if and only if

$$\underline{\theta} - \frac{p^{\overline{F}}}{1 - p^{\overline{F}}}(\overline{\theta} - \underline{\theta}) \geq 0$$

which can be rewritten as a function of the prior mean to obtain condition 1 in Assumption 1.

Henceforth, we prove the result holds for $q_L^s > 0$. Under Assumption 1, H is BMC. In this case, by

Lemma 6, there is always a $\underline{k} > 0$ such that $\operatorname{supp} \overline{F} = \operatorname{supp} F^k = \{\underline{\theta}, \overline{\theta}\}$ and $q^k = q^s$ for that distribution, for all $k \leq \underline{k}$. Let $W(k)$ and $CS(k)$ denote the welfare and consumer surplus obtained at the optimal solution, respectively. Then, notice that because the distribution and optimal quality do not change in this interval, welfare changes only to the extent that acquisition costs increase. We can then calculate the derivative of consumer surplus at zero using the fact that profits, Q are welfare minus consumer surplus:

$$\begin{aligned} CS'(0) = W'(0) - Q'(0) &= -\sum_{i \in \{L,H\}} p_i^{\overline{F}}[H(\theta_i)] + p_H^{\overline{F}}\{H(\overline{\theta}) - H(\underline{\theta}) - H'(\underline{\theta})(\overline{\theta} - \underline{\theta})\} \\ &= -H(\underline{\theta}) - p_H^{\overline{F}} H'(\underline{\theta})(\overline{\theta} - \underline{\theta}) > 0 \end{aligned}$$

where the last inequality comes from strong convexity of H and the fact that $H(\mu) = 0$. Moreover, when H is BMC, the derivative of profits and welfare is continuous for $k \leq \overline{k}$, which shows that consumers' surplus is increasing for sufficiently small k.

Consumer Surplus decreases for high k We know from Lemma 5 that there is $\overline{k} > 0$ such that $\operatorname{supp} F^k = \{\mu\}$ for all $k \geq \overline{k}$. Additionally, for k sufficiently high, we have $\theta_o^k \equiv \theta_o(\psi^k) > \underline{\theta}$. Then, by 6.

$$c'\left(kH'(\mu) - kH'(\theta_o^k)\right) = \theta_o^k$$

Differentiation then provides:

$$\frac{d\theta_o^k}{dk} = \frac{c''(q^k)\frac{q^k}{k}}{1 + c''(q^k)kH''(\theta_o^k)}$$

We can apply that in the derivative of $CS(k)$ to obtain:

$$CS'(k) = \frac{CS}{k}(k) - \frac{H''(\theta_o^k)c''(q^k)q^k}{1 + c''(q^k)kH''(\theta_o^k)}(\mu - \theta_o^k)$$

By the mean value theorem, there is $m_k \in [\theta_o^k, \mu]$ such that $\frac{CS}{k}(k) = H''(m_k)\frac{(\mu-\theta_o^k)^2}{2}$. We then have that the derivative above can be rewritten as:

$$CS' = \left(H''(m_k)\frac{(\mu - \theta_o^k)}{2} - \frac{H''(\theta_o^k)c''(q^k)q^k}{1 + c''(q^k)kH''(\theta_o^k)}\right)(\mu - \theta_o^k)$$

The first term in parentheses converges to zero, but not the second, which means that as k is large enough the whole derivative is negative. ■

Proof of Proposition 5

Recall that the distribution F to be estimated is composed of three elements: $\operatorname{supp} F = \{\theta_L, \theta_H\}$ and $1 - F(\theta_L) = p_H^F$. Denote the mean of F by μ. As argued in the text, p_H^F is correctly identified from the distribution of signed contracts and θ_H by the high quality efficiency, following Proposition 3. So we focus on the estimation of θ_L. Call this estimator $\hat{\theta}_L$. That value can be estimated using the second best contract formula. In particular:

$$q_L = \hat{\theta}_L - \frac{p_H^F}{1 - p_H^F}(\theta_H - \hat{\theta}_L)$$

Notice, however, that, under costly information acquisition, $\mathbb{E}_F[q] = \theta_o$. Assuming that q_L solves the second-best problem, we obtain:

$$\hat{\theta}_L = E_F[q] = \theta_o \leq \hat{\theta}_L$$

Therefore, the estimated mean of the distribution F is $\hat{\mu} = p_H^F\theta_H + (1 - p_H^F)\hat{\theta}_L \leq p_H^F\theta_H + (1 - p_H^F)\theta_L = \mu$. Because the expected quality is known, the estimated wedge must be weakly smaller than the real wedge. That is:

$$\hat{\tau}(F, q) = \hat{\mu} - p_H^F q_H - (1 - p_H^F)q_L \leq \mu - p_H^F q_H - (1 - p_H^F)q_L = \tau(F, q)$$

■

Proof of Proposition 6

If the principal wants to induce no information acquisition, she offers a single contract $\{q, t\}$ to be accepted by the ex-ante agent. By incentive compatibility, there exists a cutoff type, $\theta_u \leq \mu$, such that the interim payoff of any agent with posterior $\theta > \theta_u$ is $q(\theta - \theta_u)$. Because the agent can only choose between this contract or no contract, he will choose to sign the contract if he receives the high signal, θ_H, and to opt-out otherwise — that is, $\theta_L \leq \theta_u$. Thus, his information acquisition problem is:

$$\max_{\theta_L \leq \theta_u \leq \mu \leq \theta_H} \frac{\mu - \theta_L}{\theta_H - \theta_L} q(\theta_H - \theta_u) - \kappa(\theta_L, \theta_H), \tag{IA'}$$

where $\frac{\mu - \theta_L}{\theta_H - \theta_L}$ is the probability of the high signal. If acquiring no information is optimal, we abuse notation to say μ solves IA'. The problem of the principal is then:

$$\max_{q,\theta_u}\{\mu q - c(q) - q(\mu-\theta_u) : \mu \text{ solves } IA'\}$$

We first show that $\theta_u < \mu$. For a contradiction, assume $\theta_u = \mu$. In that case, consider a deviation from an uninformative information structure $\{\mu, \mu+\delta\}$ to $\{\mu-\varepsilon, \mu+\delta\}$. For sufficiently small ε, the change in agent's utility is approximately (minus) the derivative of the objective function in IA' with respect to θ_L, evaluated at $\{\mu, \mu+\delta\}$:

$$\left(q - \frac{\partial\kappa}{\partial\theta_L}(\mu, \mu+\delta)\right)\varepsilon$$

By making δ close to zero, the second term in the parentheses converges to zero by Assumption 2, whereas the first term is positive. Thus, the agent can benefit from acquiring a small amount of information. Therefore, for no information to be optimal for the consumer, $\theta_u < \mu$.

We conclude by showing that this implies underprovision. Indeed, assume for a contradiction that $c'(q^*) \geq \mu$. Notice that the objective function in the principal's problem is:

$$\theta_u q - c(q).$$

Thus, reducing q^* clearly increases the objective function, since $\theta_u < \mu$. We just need to prove that it is feasible to do so. Feasibility is equivalent to, for all $\theta_L < \mu < \theta_H$:

$$q(\mu-\theta_u) - \frac{\mu-\theta_L}{\theta_H-\theta_L}q(\theta_H-\theta_u) - \kappa(\theta_L,\theta_H) \geq 0,$$

For sufficiently small $\varepsilon > 0$, consider $q' = q^* - \varepsilon > 0$. Then:

$$\begin{aligned}
q'(\mu-\theta_u) - \frac{\mu-\theta_L}{\theta_H-\theta_L}q'(\theta_H-\theta_u) - \kappa(\theta_L,\theta_H) \geq \\
-\varepsilon\left(\mu-\theta_u-(\mu-\theta_L)\frac{\theta_H-\theta_u}{\theta_H-\theta_L}\right) = \\
-\varepsilon\frac{(\theta_H-\mu)(\theta_L-\theta_u)}{\theta_H-\theta_L} \geq 0,
\end{aligned}$$

where the first inequality follows from feasibility of q^*, the equality collects terms, and the last inequality follows from $\theta_H \geq \mu > \theta_u \geq \theta_L$.

We have then proved that q' is feasible and, for small enough ε it increases the principal's objective, contradicting optimality of q^*. Therefore, $c'(q^*) < \mu$. ■

Lemma 5. *Let F^k be the optimal information structure for cost level k. There is $\overline{k}$ such that* $\operatorname{supp} F^k = \{\mu\}$ *for all*

$k \geq \overline{k}$

Proof of Lemma 5

Recall that q's are uniformly bounded, so there is $R > 0$ with $q_H - q_L \leq R$. By M, if $\theta_H^k > \theta_L^k$ we then have:

$$kH'(\theta_H^k) - kH'(\theta_L^k) \leq R$$

Fix any $k > 0$ the previous equation implies:

$$\mu \leq \theta_H^k \leq H'^{-1}\left(\frac{R}{k} + H'(\mu)\right)$$

similarly:

$$H'^{-1}\left(-\frac{R}{k} + H'(\mu)\right) \leq \theta_L^k \leq \mu$$

Let $\tilde{k}$ large enough such that $k \geq \tilde{k}$ implies $\theta_H^k \leq \tilde{\theta}_H < \overline{\theta}$ and $\theta_L^k \geq \tilde{\theta}_L > \underline{\theta}$. This is clearly possible. It implies that, for, $k \geq \tilde{k}$, the principal can restrict attention to information structures such that $\tilde{\theta_L} \leq \theta_L \leq \mu \leq \theta_H \leq \tilde{\theta}_H$. Thus, for $k \geq \tilde{k}$, H', H'' and H''' are all bounded. We use this result next.

Recall that profits at $F, \{q_i\}, \psi$ are surplus minus interim rents:

$$\sum_{i \in \{L,H\}} p_i^F \left\{\theta_i q_i - c(q_i) - U(\theta_i)\}\right\}$$

By M, $q_i = kH'(\theta_i) + \psi$, for $k \geq \tilde{k}$. Define $\underline{F}$ with supp$\underline{F} = \{\mu\}$ $q_\mu \equiv kH'(\mu) + \psi$ and $U_\mu = kH(\mu) + \psi\mu + \max_{v \in \Theta}\{-H(v) - \psi v\}$. The choice variables $\{\underline{F}, q_\mu, U_\mu, \psi\}$ are feasible: U_μ was defined so as to satisfy TP, and M and BC are trivially satisfied.

By convexity of H, $U_\mu \leq \sum_{i \in \{L,H\}} p_i^F \{kH(\theta_i) + \psi\theta_i + \max_{v \in \Theta}\{-H(v) - \psi v\}\} < \sum_{i \in \{L,H\}} p_i^F U_i$.

Define the function: $S^k(\theta) \equiv \theta(kH'(\theta) + \psi) - c(kH'(\theta) + \psi)$. Clearly, $\sum_i p_i^F S^k(\theta_i)$ is the surplus for $\{F, \{q_i\}, \psi\}$, and $S^k(\mu)$ is the surplus for $\{\underline{F}, q_\mu, \psi\}$. We show that, for sufficiently large k, $S^k(\mu) \geq \sum_i p_i^F S^k(\theta_i)$. For that, take second derivative of S^k and divide it by $kH''(\theta)$. Because H is strongly convex, $H''(\theta) > 0$, so this can be done. For $k \geq \tilde{k}$

$$\frac{S^{k\prime\prime}(\theta)}{kH''(\theta)} = 2 + (\theta - c'(kH'(\theta) + \psi))\frac{H'''(\theta)}{H''(\theta)} - c''(kH'(\theta) + \psi)kH''(\theta) \leq M - kW$$

for some $M, W > 0$. The inequality comes from the following: first, because c and H are strongly convex, W can be found such that the last term is less than $-kW$. Then, notice that if $H''' < 0$, the middle term is negative and the inequality holds for any $M \geq 2$. If $H''' > 0$, the argument in the beginning of this proof shows that $\frac{H'''}{H''}$ is bounded, so we can take $M \geq 2 + \max_{v \in [\tilde{\theta}_L, \tilde{\theta}_H]} \left\{ \frac{H'''(v)}{H''(v)} \right\}$. Then, by picking $k > \max\{\tilde{k}, \frac{M}{W}\}$ we have that S^k is concave, thus, $S^k(\mu) \geq \sum_i p_i^F S^k(\theta_i)$, by Jensen's inequality.

Because the surplus is higher and the rents are lower, we proved that $\{\underline{F}, q_\mu, U_\mu, \psi\}$ dominates the initial contract for high k. Since this is also a feasible menu, we have proved that the solution must have no information acquisition for high enough k. ■

Lemma 6. *Let H be BMC and F^k be the optimal information structure for cost level k. There is $\underline{k} > 0$ such that for $k \leq \underline{k}$,* $\operatorname{supp} F^k = \{\underline{\theta}, \overline{\theta}\}$

Proof of Lemma 6

We proceed in 3 steps. We first show that $\theta_H = \overline{\theta}$ for small enough k. We then prove that $\theta_L = \theta_o$, and finally we conclude that $\theta_o = \underline{\theta}$.

Step 1 We start by showing that, for low enough k, $\theta_H = \overline{\theta}$. Start defining the surplus function $S(\theta, q) = \theta q - c(q)$, and note that this function is strictly increasing in θ for $q > 0$. Let $c'(q(\mu)^{\mathrm{f}}) = \mu$ define the efficient quality for type μ. We fix $\varepsilon > 0$ such that:

$$S(\overline{\theta}, \overline{q}^{\mathrm{f}}) - S\left(\overline{\theta}, q(\mu)^{\mathrm{f}} + \varepsilon\right) > 2\varepsilon,$$

which is possible because $q(\mu)^{\mathrm{f}} < \overline{q}^{\mathrm{f}}$, and $\overline{q}^{\mathrm{f}}$ uniquely maximizes the continuous surplus function at $\overline{\theta}$.

Now, recall that we have proved ψ is bounded and choose $\underline{k}$ small enough such that:

$$k\left(H(\overline{\theta}) + \psi + \max_{[\underline{\theta}, \overline{\theta}]}\{-H(v) - \psi v\}\right) \leq \varepsilon,$$

and

$$\max\{\overline{\theta}, 1\} k \left(H'(\overline{\theta}) - H'(\underline{\theta})\right) \leq \varepsilon,$$

which is possible by BMC. We are then ready to prove our result. Choose $k \leq \underline{k}$, and let $\operatorname{supp} F = \{\theta_L, \theta_H\}$, with $\theta_H < \overline{\theta}$ and $p = \frac{\mu - \theta_L}{\theta_H - \theta_L}$ being the probability of the high signal. Denote by π_i, $i \in \{L, H\}$ the profits accrued with type i. Consider an alternative information structure for the principal, with $\theta'_H = \overline{\theta}$, $\theta'_L = \theta_L$ and $p' = \frac{\mu - \theta_L}{\overline{\theta} - \theta_L}$ being the probability of the high signal. In that case, M allows $q'_H = \overline{q}^{\mathrm{f}}$. The gain from this deviation is:

$$
\begin{aligned}
&p'\left(S(\overline{\theta},\overline{q}^{\mathrm{f}})-k\left(H(\overline{\theta})+\psi+\max_{[\underline{\theta},\overline{\theta}]}\{-H(v)-\psi v\}\right)-\pi_H\right)-(p-p')(\pi_H-\pi_L)\\
&=\frac{\mu-\theta_L}{\overline{\theta}-\theta_L}\left(S(\overline{\theta},\overline{q}^{\mathrm{f}})-\pi_H-k\left(H(\overline{\theta})+\psi+\max_{[\underline{\theta},\overline{\theta}]}\{-H(v)-\psi v\}\right)-\frac{\overline{\theta}-\theta_H}{\overline{\theta}-\theta_L}(\pi_H-\pi_L)\right)\\
&\geq\frac{\mu-\theta_L}{\overline{\theta}-\theta_L}\left(\underbrace{S(\overline{\theta},\overline{q}^{\mathrm{f}})-\pi_H}_{(a)}-\underbrace{k\left(H(\overline{\theta})+\psi+\max_{[\underline{\theta},\overline{\theta}]}\{-H(v)-\psi v\}\right)}_{(b)}-\underbrace{\pi_H-\pi_L}_{(c)}\right),
\end{aligned}
$$

where the equality follows from substituting p and p' by its respective expressions, and the inequality follows from $\frac{\overline{\theta}-\theta_H}{\overline{\theta}-\theta_L}\leq\frac{\overline{\theta}-\mu}{\overline{\theta}-\theta_L}\leq\frac{\overline{\theta}-\mu}{\overline{\theta}-\mu}=1$.

We prove the term in parentheses is positive. Indeed, because $\theta_H<\overline{\theta}$, M implies:

$$q(\theta_H)\leq q(\theta_L)+k\left(H'(\theta_H)-H'(\theta_L)\right)\leq q(\mu)^{\mathrm{f}}+k\left(H'(\overline{\theta})-H'(\underline{\theta})\right)\leq q(\mu)^{\mathrm{f}}+\varepsilon.$$

Thus, for (a):

$$S(\overline{\theta},\overline{q}^{\mathrm{f}})-\pi_H\geq S(\overline{\theta},\overline{q}^{\mathrm{f}})-S\left(\overline{\theta},q(\mu)^{\mathrm{f}}+\varepsilon\right)>2\varepsilon.$$

For (c), we have:

$$\pi_H-\pi_L=\int_{\theta_L}^{\theta_H}(\theta-c'(q(\theta)))kH''(\theta)\leq\overline{\theta}k(H'(\overline{\theta})-H'(\underline{\theta}))\leq\varepsilon.$$

Furthermore, $k\leq\underline{k}$ guarantees (b) is bounded by ε. Thus, the gain from the alternative information structure is positive, and we proved $\theta_H=\overline{\theta}$.

Step 2. We now show $\theta_L=\theta_o$ for small enough k. For that, we will consider how profits change going from any $\operatorname{supp}F=\{\theta,\overline{\theta}\}$ to $\operatorname{supp}F'=\{\theta',\overline{\theta}\}$, $\theta'>\theta$. Let p and p' be the probabilities of the high signal, π_i and π_i' the profits with type i in F and F' respectively. Note that, in standard screening, second-best profits are increasing with type. This implies:

$$\min_{\theta_L\leq\mu}\left\{S(\overline{\theta},\overline{q}^{\mathrm{f}})-q^s(\theta_L)(\overline{\theta}-\theta_L)-S(\theta_L,q^s(\theta_L))\right\}>0,$$

Recalling that $\psi=q_L-kH'(\theta_L)$, and that $q_L\leq q^s(\theta_L)$ we can choose a small enough ε and k such that, for any F:

$$
\begin{aligned}
&\pi_H - \pi_L \\
&= S(\overline{\theta},\overline{q}^{\mathrm{f}}) - S(\theta_L, kH'(\theta_L)+\psi) - k\Big(H(\overline{\theta}) - H(\theta_L)\Big) - \psi(\overline{\theta}-\theta_L) \\
&= S(\overline{\theta},\overline{q}^{\mathrm{f}}) - S(\theta_L, kH'(\theta_L)+\psi) - k\Big(H(\overline{\theta}) - H(\theta_L) - H'(\theta_L)(\overline{\theta}-\theta_L)\Big) - q_L(\overline{\theta}-\theta_L) \\
&\geq S(\overline{\theta},\overline{q}^{\mathrm{f}}) - S(\theta_L, q^s(\theta_L)) - q^s(\theta_L)(\overline{\theta}-\underline{\theta}) - k\Big(H(\overline{\theta}) - H(\underline{\theta}) - H'(\underline{\theta})(\overline{\theta}-\underline{\theta})\Big) \geq \varepsilon.
\end{aligned}
$$

Further, choose k small enough such that:

$$
\overline{\theta}k \max_{\theta\in[\underline{\theta},\overline{\theta}]} \{H''(\theta)\}(\overline{\theta}-\underline{\theta})(\overline{\theta}-\mu) < \varepsilon/2,
$$

which is possible due to boundedness of H'' by BMC. Then, recalling that $\theta_H = \overline{\theta}$, the difference between profits with F' and F is:

$$
\begin{aligned}
&(p'-p)(\pi_H-\pi_L) + (1-p')(\pi_L' - \pi_L) \\
&= -\frac{(\overline{\theta}-\mu)}{(\overline{\theta}-\theta)(\overline{\theta}-\theta')}(\theta'-\theta)(\pi_H-\pi_L) + (1-p')\int_\theta^{\theta'} (\theta - c'(kH'(\theta)+\psi))\,kH''(\theta)d\theta \\
&\leq -\frac{(\overline{\theta}-\mu)}{(\overline{\theta}-\theta)(\overline{\theta}-\theta')}(\theta'-\theta)(\pi_H-\pi_L) + (1-p')\overline{\theta}k \max_{\theta\in[\underline{\theta},\overline{\theta}]}\{H''(\theta)\}(\theta'-\theta) \leq -\frac{\varepsilon}{2}(\theta'-\theta),
\end{aligned}
$$

where the first equality comes from rewriting p and p' by Bayesian consistency, and from the definition of profits, with $q_L(\theta) = kH'(\theta)+\psi$. We have then proved that choosing a $\theta' > \theta$ is always worse than choosing θ itself. Thus, $\theta_L = \theta_o$.

Step 3. Now we prove $\theta_o = \underline{\theta}$. We do it in two cases. Let $\underline{q}^s$ be the second-best contract for the lowest type, $\underline{\theta}$, under full information. We start with the case of $\underline{q}^s > 0$. Let p_0 be the probability of the high signal under full information. We have:

$$
\underline{q}^s = \underline{\theta} - \frac{p_0}{1-p_0}(\overline{\theta}-\underline{\theta}) > 0,
$$

and we obtain $p_0\overline{\theta} < \underline{\theta}$. Define $\underline{\theta} - p_0\overline{\theta} = \varepsilon$. Further, Choose k small enough such that:

$$
c'\Big(k\Big(H'(\overline{\theta}) - H'(\underline{\theta})\Big)\Big) \leq \frac{\varepsilon}{2}.
$$

Assume now $\theta_o > \underline{\theta}$. By definition of θ_o we must have $-H'(\theta_o) = \psi$, and, therefore $q_L = kH'(\theta_L) - kH'(\theta_o) < k\Big(H'(\overline{\theta}) - H'(\underline{\theta})\Big)$.

Then:

$$
\theta_o = p\overline{\theta} + (1-p)c'(q_L) \leq p\overline{\theta} + \frac{\varepsilon}{2} \leq p_0\overline{\theta} + \frac{\varepsilon}{2} = \underline{\theta} - \frac{\varepsilon}{2} < \underline{\theta},
$$

which is a contradiction. The first equality is by the first-order condition for ψ, and the fact that $\theta_H = \overline{\theta}$ and is served efficiently, the first inequality follows from the constraint on k above, and the second inequality by Bayesian consistency, since $\theta_L \geq \underline{\theta}$ implies $p \leq p_0$.

We conclude with the case of $\underline{q}^s = 0$. Note that M coupled with Step 2 imply that, for any $\theta_o > \underline{\theta}$, we must have $q_L = 0$, since $\theta_L = \theta_o$. Thus, if $\theta_L = \theta_o > \underline{\theta}$, the principal maximizes:

$$p\Big(S(\overline{\theta},\overline{q}^{\mathrm{f}}) - k\Big(H(\overline{\theta}) - H(\theta_L) - H'(\theta_L)(\overline{\theta} - \theta_L)\Big)\Big)$$

Thus, following the argument in Step 2, by choosing k sufficiently small, we can obtain that the derivative of the function above in θ_L satisfies, for all θ_L:

$$-\frac{(1-p)}{\overline{\theta} - \theta_L}\pi_H + (1-p)kH''(\theta_L) < 0,$$

so $\theta_L = \underline{\theta}$. ■

Appendix B: The Revelation Principle

In this section we show that an appropriate version of the revelation principle holds in our framework, even though information is endogenous. No assumption in the acquisition cost function is required. The proof works in two steps. First, given an information structure F, the usual revelation principle shows us that any equilibrium can be obtained by a direct mechanism on $\operatorname{supp} F$, the endogenous type set. Then, we show that this direct mechanism can be extended to the set of all possible types, Θ, without affecting information acquisition decisions. We prove a more general version of the revelation principle, for an arbitrary number of finite states.

Let $\Sigma = \{\vartheta_1, ..., \vartheta_N\}$ be the set of states. We define the primitive game as: $\hat{\mathcal{G}} = (\Sigma, \Pi, K)$, where $\Pi \in \Delta(\Sigma)$ is the prior and $K : \Delta(\Theta) \to \mathbb{R}_+$ are acquisition costs, where $\Theta = [\underline{\theta}, \overline{\theta}]$. An information structure is a distribution over posterior means $F \in \Delta(\Theta)$. The set of feasible information structures is $\mathcal{P}_\Pi$. In our problem, this is the Bayesian consistent set of information structures. For the proof, the only relevant aspect of this set, is that for any $A \subseteq \Theta$, there is $F \in \mathcal{P}_\Pi$ with $A \subset \operatorname{supp} F$. This is satisfied by the Bayesian consistency constraint for any number of states. Finally, a mechanism $\Gamma = (M, g)$ is an arbitrary message set, M and an outcome function $g : \mathcal{M} \to \mathbb{R}^2_+$, $g(m) = \big(q(m), t(m)\big)$.

An equilibrium for a mechanism Γ is a pair of information structure and messaging strategy (F, m), $F \in \mathcal{P}_\Pi$, $m : \operatorname{supp} F \to \mathcal{M}$ such that:

1. Given F, m is an optimal strategy for the agent in the mechanism. That is, for all $\theta \in \operatorname{supp} F$:

$$\theta q(m(\theta)) - t(m(\theta)) \geq \theta q(\omega) - t(\omega)$$

   for all $\omega \in \mathcal{M}$

2. F is optimally chosen:

$$\mathbb{E}_F\Big[\theta q(m(\theta)) - t(m(\theta))\Big] - K(F) \geq \mathbb{E}_G\Big[\sup_{\omega \in M}\{\theta q(\omega) - t(\omega)\}\Big] - K(G)$$

   for all $G \in \mathcal{P}_\Pi$

A particular instance of a mechanism is a direct mechanism. Usually, the direct mechanism has a natural type space to be defined as it's message space, but here, as types are endogenous, this is not the case. We define a direct mechanism to be (Θ, g). We say that an equilibrium from a direct mechanism satisfies truthful revelation if the optimal strategy is the identity, $Id : \Theta \to \Theta$. With this definition, we have the following revelation principle, that justifies our focus on direct mechanisms over Θ in the main text.

Theorem 1 (Revelation Principle)**.** *Fix a game $\mathcal{G}$. If (F, m) is an equilibrium for the mechanism Ω, then for some direct mechanism Γ there is an equilibrium, (F, Id), with (1) truthful revelation and (2) same payoffs and allocations:.*

Proof of Theorem 1

Let (F, m) be the equilibrium for mechanism Γ. Start building a direct mechanism. Given F, part 1 of the equilibrium definition implies that m is a Bayesian Nash Equilibrium in $\operatorname{supp} F$. The standard revelation principle then implies that $\tilde{\Gamma} = (\operatorname{supp} F, g \circ m)$ is a direct mechanism on $\operatorname{supp} F$ with the same equilibrium.

We now extend the mechanism $\tilde{\Gamma}$. Define $h = g \circ m$ for $\theta \in \operatorname{supp} F$. If $\theta \notin \operatorname{supp} F$, consider the problem:

$$\sup_{v \in \operatorname{supp} F} \theta q(v) - t(v) \tag{17}$$

We now prove that the problem above has a solution. Otherwise, it would be unbounded. Define $U(v) = vq(v) - t(v)$ for $v \in \operatorname{supp} F$. If the objective in equation equation 17 is unbounded for some θ, then there is a sequence v_n in $\operatorname{supp} F$ such that the objective is always larger than n when evaluated in this sequence. Rewriting the objective, we have $U(v_n) + (\theta - v_n) q(v_n)$. First, see that $U(v_n)$ is bounded. Otherwise, up to a subsequence, we have:

$$n < v_n q(v_n) - t_n \leq (\max \operatorname{supp} F) q(v_n) - t_n$$

where the second inequality comes from q being non-negative, and $\max \operatorname{supp} F$ is well defined as supports are closed sets, Θ is compact and $\operatorname{supp} F \subset \Theta$. Then, that would imply that the optimization has no solution for $\max \operatorname{supp} F$ in the limit, which is a contradiction with the original equilibrium.

If $U(v_n)$ is bounded, then $U(v_n) + (\theta - v_n)q(v_n)$ can only be unbounded, if $q(v_n)$ grows indefinitely and $\theta > v_n$ for all sufficiently large n. On top of that, notice that $v_n \nrightarrow \theta$, as $\operatorname{supp} F$ is closed. Therefore, there exists a neighborhood of θ, N such that $\theta' \in N$ also achieves unbounded values for the objective in equation 17. Consider a distribution $G \in \mathcal{P}_\Pi$ such that $N \in \operatorname{supp} G$. This distribution exists by assumption. Then, under the original mechanism Ω, and for large enough n:

$$\begin{aligned} \mathbb{E}_F\Big[\theta q(m(\theta)) - t(m(\theta))\Big] - K(F) < n < \\ \int_{\theta \notin N} \Big[\sup_{\omega \in M} \{\theta q(\omega) - t(\omega)\}\Big] dG + \int_{\theta \in N} (\theta q(m(v_n)) - t(m(v_n))) dG - K(G) \end{aligned}$$

which is a contradiction to (F, m) being an equilibrium in the original mechanism. This implies the problem in equation 17 has a solution. Let $v(\theta)$ be a solution for that problem. Then, define $h(\theta) = h(v(\theta))$, for $\theta \notin \operatorname{supp} F$. Consider the direct mechanism $\Gamma = (\Theta, h)$. In words, this direct mechanism selects, for any type in $\operatorname{supp} F$, their equilibrium allocation, and for any type not in $\operatorname{supp} F$ their favorite allocation among those given to types in $\operatorname{supp} F$.

Because $\tilde{\Gamma} = \Gamma|_{\operatorname{supp} F}$, it is clear that the first condition of equilibrium is satisfied by (F, Id). For the second condition:

$$\begin{aligned} \mathbb{E}_F\Big[\theta q(\theta) - t(\theta)\Big] - K(F) \\ = \mathbb{E}_F\Big[\theta q(m(\theta)) - t(m(\theta))\Big] - K(F) \\ \geq \mathbb{E}_G\Big[\sup_{\omega \in M} \{\theta q(\omega) - t(\omega)\}\Big] - K(G) \\ \geq \mathbb{E}_G\Big[\sup_{\omega \in \{z \in M : m(v) = z, v \in \operatorname{supp} F\}} \{\theta q(\omega) - t(\omega)\}\Big] - K(G) \\ = \mathbb{E}_G\Big[\sup_{\theta \in \Theta} \{\theta q(\theta) - t(m(\theta))\}\Big] - K(G) \end{aligned}$$

Where the first equality uses the definition of h; the first inequality uses the second part of the definition of equilibrium for the original equilibrium (F, m); the second inequality is just a consequence of a supremum being taken over a smaller set; and the second equality is due, again, to the definition of h. Hence, both conditions of equilibrium are satisfied by truthtelling. The allocations are, of course, the same, since F is the same and $\tilde{\Gamma}|_{\operatorname{supp} F} = \Gamma$. ∎

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